

What does the 'b' do?

Maths: Graphs of linear and quadratic functions

Age: 13 and up

The premise of this activity is that students will have had some experience with linear graphs and be ready to extend what they know to quadratic graphs. There is a curious inconsistency in English teaching such that the coefficients of a polynomial of order one are labelled as m and c , whereas for all other orders, coefficients are labelled a, b, c , etc. as required. When looking at polynomials of different orders, this can only be confusing to learners and hence here we will notate a general linear function as $f(x) = ax + b$. If you would prefer to translate to m and c , then feel free!

A polynomial with variable x is a function made up as a sum of multiples of increasing powers of x . The order of the polynomial is the highest power of x .

We explore the effect on the graph of different polynomials of changing the coefficients of the powers of x .

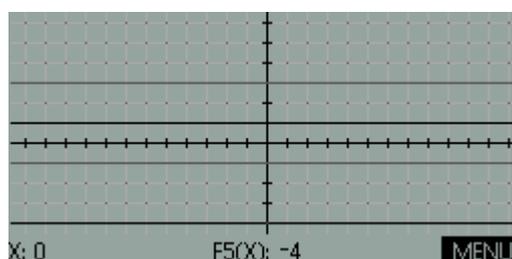
		a	b	c	d
Constant	$f(x) = a$				
Linear	$f(x) = ax + b$				
Quadratic	$f(x) = ax^2 + bx + c$				
Cubic	$f(x) = ax^3 + bx^2 + cx + d$				

We use 'Math Gym' ¹ as our assessment for learning mechanism.

The class stands up and holds their arms out to represent the given function. ²

Start with constant functions: $f(x) = 1$, then $f(x) = 3$, then $f(x) = 8$ (some standing on chairs is needed!), then $f(x) = -1$, then $f(x) = -4$ etc.

If needs be examples can be plotted, the first one by hand, then on the HP39gII emulator.



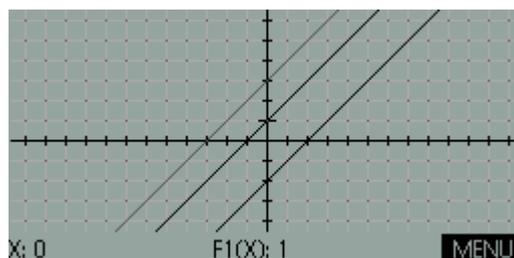
Students can now fill in their table for 'what does the 'a' do in a constant function.

¹ Shown to me by Professor Adrian Oldknow with thanks.

² Notice that I prefer to refer to functions as $f(x)$ from the start as the status of the variable y is ambiguous, however, substitute $y =$ if you disagree.

Now move on to linear functions: $f(x) = x$, then $f(x) = 2x$, then $3x$, then $0.5x$, then $-x$, etc. Now

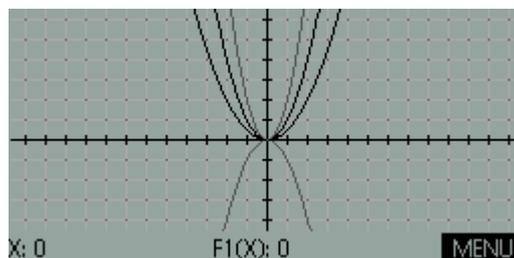
$f(x) = x + 1$, then $x + 3$, then $x - 2$, etc. Then $f(x) = 3x + 1$, then $3x - 2$ etc. Again we can test with a hand plotted example followed by any number on the emulator.



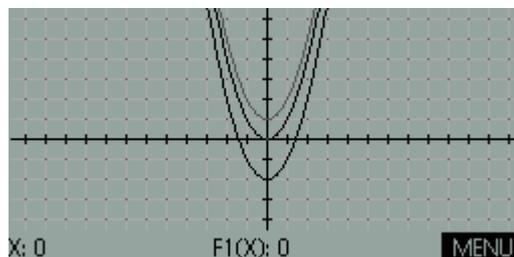
Students can now fill in their table for 'what do the 'a' and the 'b' do in a linear function.

We now move on to quadratic functions:

First: $f(x) = x^2$, then $f(x) = 2x^2$ (you may need to check this one with the emulator, as many are apt to expect the graph to get wider. Why it does not is an excellent discussion point). Then $f(x) = 3x^2$, then $f(x) = 0.5x^2$, then $f(x) = -x^2$ (this last one can be painful if not done with care!)



Second: $f(x) = x^2$ (to reset, as it were). Then $f(x) = x^2 + 1$, then $f(x) = x^2 + 3$, then $f(x) = x^2 - 2$, etc.



So, with a little practice, everyone knows what the 'a' and the 'c' do.

Finally: fix $a = 1$, and $c = 0$ and do $f(x) = x^2$ to reset, then $f(x) = x^2 + x$, then $f(x) = x^2 + 2x$, then $f(x) = x^2 + 3x$, then $f(x) = x^2 - x$, etc. I have found that a couple of people make a guess (generally moving left for the first ones and right for the negatives) and others follow. (This applies to groups of senior maths teachers as much as to school classes). In other words, no-one knows what the 'b' does. This is a striking realisation after so much confidence up to this point and clearly motivates finding out.

Students take a handheld and start the function app. (APPS/Function/F2(Reset)/F6(OK)/F6(Start))

Enter: $F1(x) = x^2$, then $F2(x) = x^2 + x$, then $F3(x) = x^2 + 2x$, etc.

Students continue to explore until a clear view is reached of exactly how the quadratic moves as 'b' varies. Quicker students can be challenged to find the function which gives the movement of the apex of the graph. Finally, this function should be extended to take account of 'a' and 'c' varying as well. It is important for the teacher to be involved in the unfolding conclusion, hence no solutions are given here.

The work can be extended to cubics and further although this may remain incomplete!

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