



Introduction

HP Prime is the latest advanced graphing calculator from Hewlett-Packard. It incorporates a full-color, multi-touch screen and comes pre-loaded with 18 HP Apps. HP Prime has a Home view, with a history of your numerical calculations, as well as a CAS view, with a history of your symbolic computations.

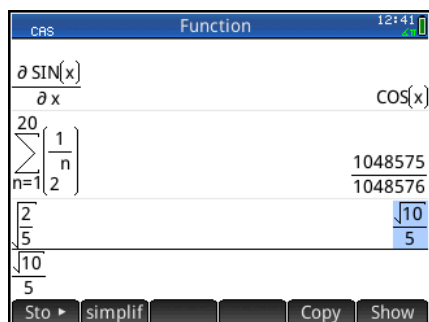
HP Prime has three design principles:

- Provide a high degree of mathematical fidelity across multiple mathematical representations
- Give students the touch-active experiences with mathematics that they have come to expect from display technologies
- Deliver a simple and seamless experience for mathematical exploration and problem-solving

Home and CAS

With HP Prime, you can choose whether to operate numerically in the Home view or symbolically using the CAS in the CAS view. For example, press  and enter $\sqrt{8}$ in the Home view to see 2.828... or press  and enter the same expression in the CAS view to see $2\sqrt{2}$.

CAS View



Calculation History

- Tap to select an entry
- Tap twice to copy it to the command line

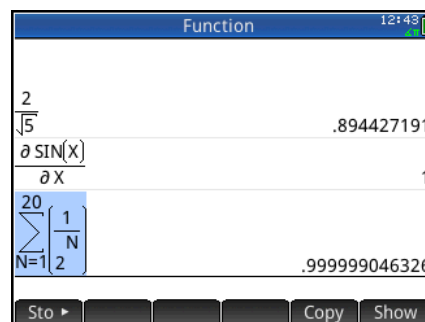
Command Line Editor

- 2D textbook format








Menu Buttons



- Tap to activate
- Context-Sensitive

Home View



Things you can do in both CAS and Home views:


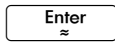


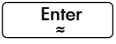

- Tap an item to select it or tap twice to copy it to the command line editor
- Tap and drag up or down to scroll through the history of calculations
- Press  to retrieve a previous entry or result from the other view
- Press the Toolbox key () to see the Math and CAS menus as well as the Catalog
- Press  to open a menu of easy-to-use templates
- Press  to exit these menus without making a selection
- Tap , , and  menu buttons once to activate

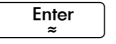
RPN Note: HP Prime supports RPN entry. In Home Settings ( ) , navigate to **Entry** and select RPN. These workshop materials use the default Textbook setting.

Example: Infinite Series

Suppose we wish to explore the series $1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \frac{9}{16} \dots$









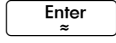
To begin with, we can compute the first few partial sums.

1. Press  to enter the Home view.
2. Type 1 and press .
3. Press  3  2 .
4. Press  to cycle through its decimal, proper fraction, and improper fraction forms
5. Continue as shown to the right until you have the first 5 partial sums

Another approach is to use lists. In the figure to the right, the List L1 is first defined to contain the 1st partial sum (1), followed by the numerator and denominator of the following term. The second line adds the next term to the sum, then adds 2 to the numerator and multiplies the denominator by 2, and stores the result back into list L1. From there, just pressing  generates the partial sums.

To use the CAS, we observe that the sum can be rewritten symbolically as

$$1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \frac{9}{16} \dots = \sum_{n=0}^{\infty} \frac{1+2n}{2^n}$$

1. Press  to open the CAS view.
2. Press  to open the template menu and select the summation template
3. To enter $n=0$, press     0
4. To enter $+\infty$ at the top, tap on the top and press , then  9 and tap on ∞
5. Tap on the right template box and enter the rational expression.
6. When you are done, press .

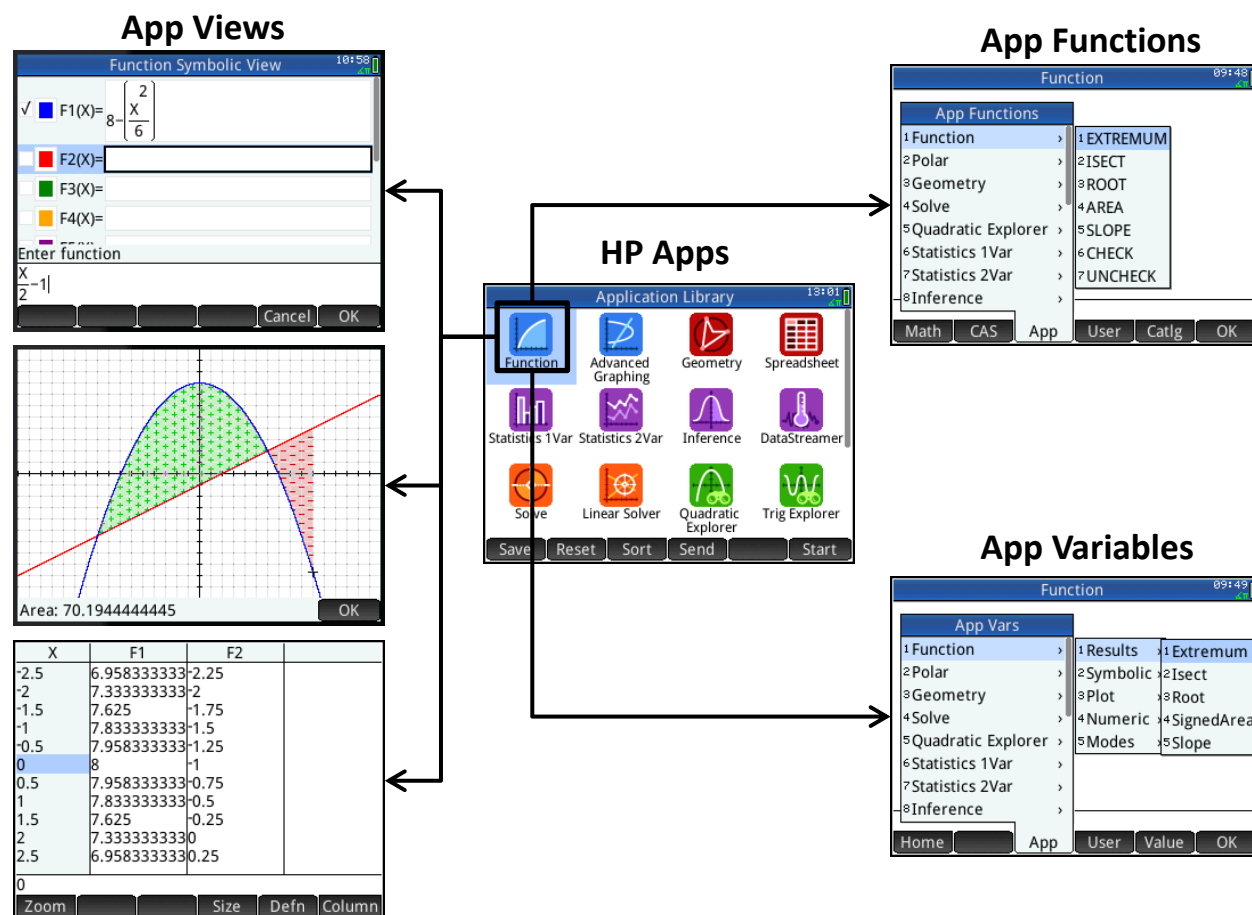
Statistics 2Var		10:45
1	1	
Ans+ $\frac{3}{2}$	$\frac{5}{2}$	
Ans+ $\frac{5}{4}$	$\frac{15}{4}$	
Ans+ $\frac{7}{8}$	$\frac{37}{8}$	
Ans+ $\frac{9}{16}$	$\frac{83}{16}$	
Sto ►		

Function		11:59
{1,3,2}►L1	{1,3,2}	
{L1{1}+ $\frac{L1{2}}{L1{3}}$, L1{2}+2, L1{3}*2}►L1	{2.5,5,4}	
	{3.75,7,8}	
	{4.625,9,16}	
	{5.1875,11,32}	
	{5.53125,13,64}	
	{5.734375,15,128}	
	{5.8515625,17,256}	
	{5.91796875,19,512}	
	{5.955078125,21,1024}	
Sto ►		

CAS		11:51
$\sum_{n=0}^{+\infty} \frac{1+2*n}{2^n}$		
Sto ►	simplif	

HP Apps

HP Apps are designed to explore mathematical topics or solve problems. All HP Apps have a similar structure, with numeric, graphic and symbolic views to make them easy to learn and easy to use. Fill the app with data while you work, and save it with a name you'll remember. Then reset the app and use it for something else. You can come back to your saved app anytime—even send it to your colleagues! HP Apps have app functions as well as app variables; you can use them while in the app, or from the CAS view, Home view, or in programs.



Press **Apps Info** to see the App Library. Drag with your finger to browse the library, then tap the icon of the app you want to use. The HP Apps are color-coded for easy identification:

- **5 graphing apps (blue)** to explore graphs—including the new Advanced Graphing App!
- **2 Special apps (red)**: the Geometry app and the Spreadsheet app
- **4 Statistics Apps (purple)** for descriptive and inferential statistics and data collection
- **4 Solver Apps (orange)** for solving specific types of problems (triangles, finance, etc.)
- **3 Explorer Apps (green)** for investigating a function's equation and its graph

The Function App

The Function App gives you all the tools you need to explore the properties of functions, including plotting their graphs, creating tables of values, and finding roots, critical points, etc.

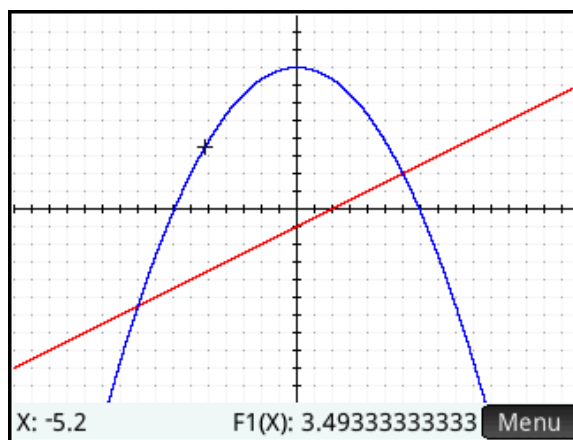
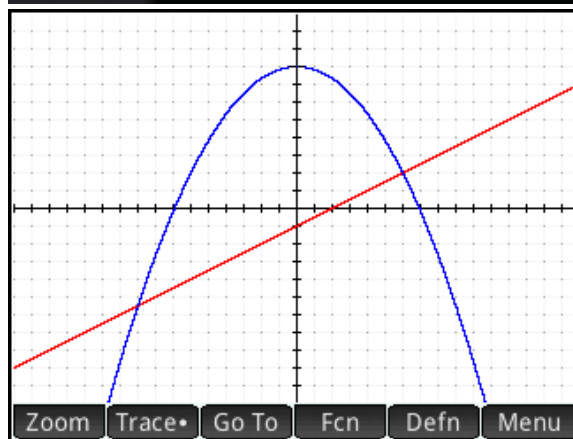
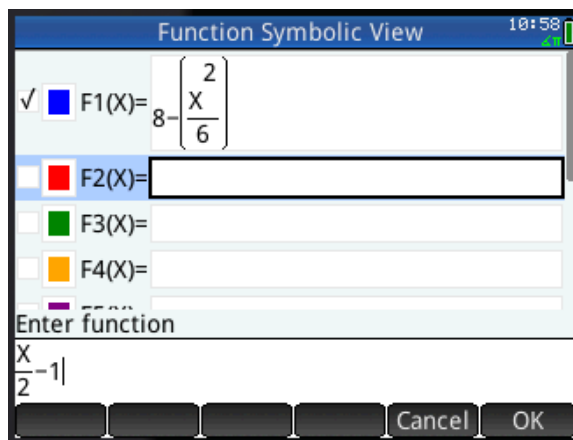
1. Press **Apps** and tap on the Function icon.
The app opens in Symbolic view.
2. Enter $8 - \frac{X^2}{6}$ in F1(X)
3. Enter $\frac{X}{2} - 1$ in F2(X)
4. For each function, tap on the color picker to choose a color and check/uncheck it to select/deselect it for graphing
5. Press **Plot** to see the graphs of your checked functions

In Plot view, tap **Menu** to open the menu. The menu buttons are:

- **Zoom**: opens the Zoom menu
- **Trace**: toggles tracing off and on
- **Go To**: enter a specific x -value and the tracer will jump to it
- **Fcn**: opens a menu of analytic functions
- **Defn**: displays the definition of a function
- **Menu**: opens and closes the menu

Things you can do:

- Press **◀** or **▶** to see that you are currently tracing on F1(X)
- Tap anywhere on the display and the tracer will jump to the x -value indicated by your finger tap while still remaining on the function being traced.
- Press **▼** or **▲** to jump from function to function for tracing
- Tap and drag to scroll the graphing window
- Press **Ans** **+** and **Base** **-** to zoom in and out on the cursor



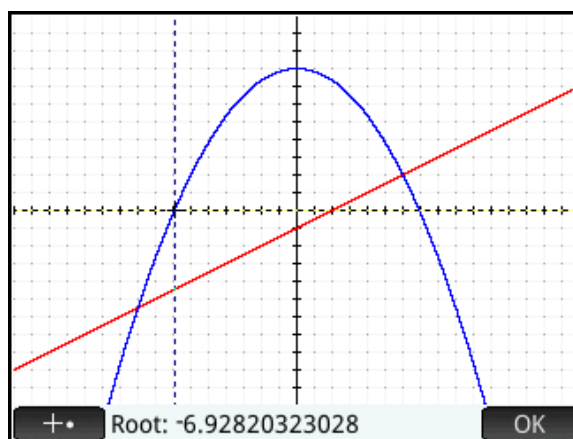
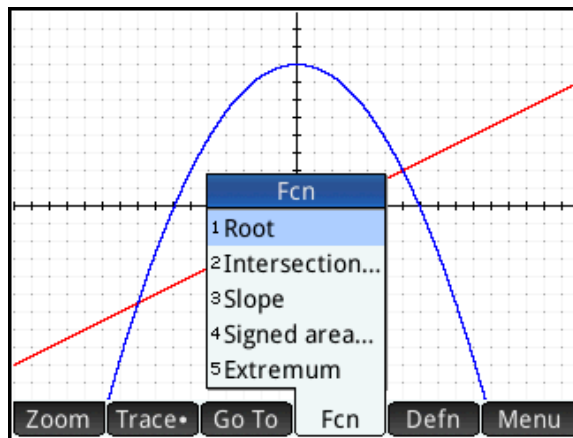
The Fcn Menu

In the following examples, we will use the options in the **Fcn** menu to explore our two functions.

Roots

First, we will find one of the roots of our quadratic function, F1(X).

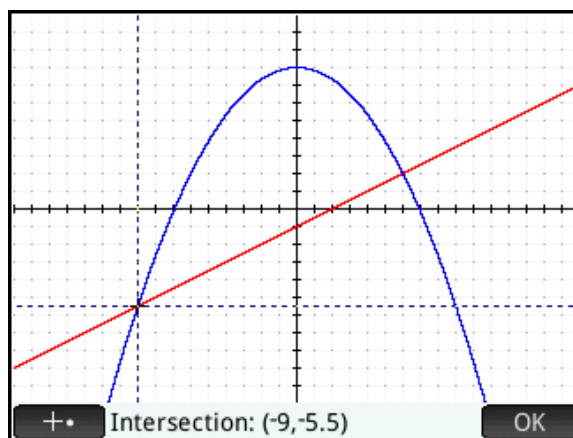
1. Tap anywhere near the left-most root of the quadratic (around $x=-7$)
2. Tap **Menu** to open the menu (if necessary)
3. Tap **Fcn** to open the **Fcn** menu
4. From the list, select **Root**, either by tapping on it, using the direction keys, or pressing **1** **Program** **Y**.
5. The value of the root ($x=-6.928\dots$) is displayed
6. Press **+** to show the exact location of the cursor and **OK** to exit



Intersection

We will now find the left-most point of intersection of the two graphs.

1. With the cursor still at the root from the previous example, tap **Fcn** to open the **Fcn** menu and select **Intersection**
2. A pop-up menu gives you the choice of finding the intersection with F2(X) or the x-axis. Press **OK** to select F2(X).
3. The intersection is displayed as shown to that right



Slope

The **Slope** option in the **Fcn** menu works similarly to **Root**, except that the slope continues to be displayed as you trace the function, until **Cancel** is pressed.

Signed Area

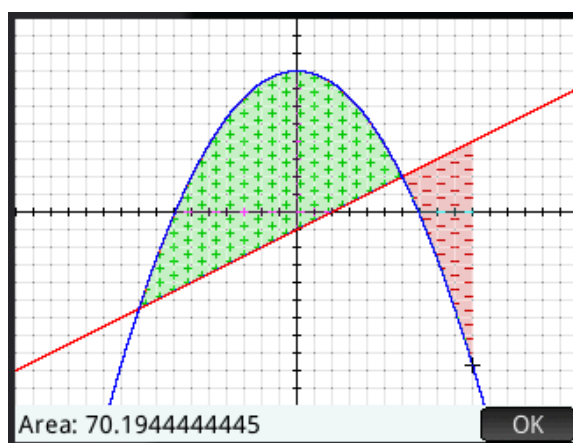
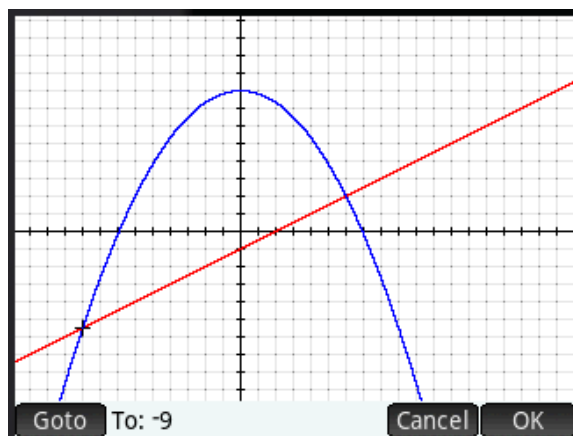
Suppose you wish to find the signed area between the curves, from $x=-9$ to $x=10$.

1. Tap **Fcn** to open the Functions menu and select Signed Area...
2. Tap near $x=-9$, use the direction keys to move the cursor to $x=-9$ exactly, and tap **OK**.
3. Now tap near $x=10$ and use the direction keys to move the cursor to $x=10$ exactly

With the touch display, navigation is improved and the experience is more interactive.

As you move the cursor, the area between the curves is filled in graphically. The color display shows you which regions have positive area and which have negative area. The fill patterns have “+” and “-” in them to remind the students that the area is signed.

4. Tap **OK** to see the area; tap **OK** again to exit.



Extremum

The Extremum option works in a manner similar to the way Root works.

The Function App: App Functions and App Variables

The five functions from the **Fcn** menu are available to you from the Home view and they store their last results in variables named after the functions. For example, in the Home view, $\text{ROOT}(\text{F1}(\text{X}), -7)$ will now return $-6.928\dots$ and that value will be stored in the app variable Root. The table below lists the most common app functions and app variables for the Function App.

Function App: The App Functions and App Variables			
Fcn Option	App Function Name and Syntax	Example	Stores results in
Root	$\text{ROOT}(\text{Expr1}, \text{Value})$	$\text{ROOT}(\text{X}^2 - 1, 0.5)$	Root
Intersection	$\text{ISECT}(\text{Expr1}, \text{Expr2}, \text{Value})$	$\text{ISECT}(\text{F1}(\text{X}), 3 - \text{X}, 2)$	Isect
Slope	$\text{Slope}(\text{Expr1}, \text{Value})$	$\text{SLOPE}(\text{X}^2 - 6, 3)$	Slope
Signed Area...	$\text{AREA}(\text{Expr1}[\text{Expr2}], \text{Val1}, \text{Val2})$	$\text{AREA}(\text{F1}(\text{X}), -6.9, 6.9)$	SignedArea
Extremum	$\text{EXTREMUM}(\text{Expr}, \text{Value})$	$\text{EXTREMUM}(\text{F2}(\text{X}), 3)$	Extremum

1. Press **Symb** **↔ Setup** to return to Symbolic view.
We will now look at some of the functionality in the Numeric view of the app.
2. Press **Shift** **Esc** **Clear** to delete all the function definitions. You will be asked to confirm this action. Tap **OK**.
3. In F1(X), enter $\frac{X^2 - 4}{X - 2}$
4. Press **Shift** **Num** **↔ Setup** to see the Numeric Setup view of the app. Change the options as shown in the figure to the right.

Note the new menu button: **PLOT→**. If pressed, this button changes the options in this view to match the settings in the Plot view. For example, with the default Plot view, Num Start would be set to -15.9 and Num Step would be set to 0.1. Tracing along the graph in plot view would then mirror navigating through the table: both would show the same (x,y) ordered pairs.

5. Press **Num** **↔ Setup** to see the Numeric view. The menu buttons are:
 - **Zoom**: same as the Plot view menu
 - **Size**: chooses a font size
 - **Defn**: displays the column definition
 - **Column**: chooses 1-4 columns
6. With any value in the x-column selected, type 2 to jump to that value.
7. Now press **Ans** **+** and **Base** **-** to zoom in and out on that row of the table just as you did to zoom in and out on the cursor in the Plot view.

Function Num Setup 17:02

Num Start: 1

Num Step: 0.1

Num Zoom: 10

Num Type: Automatic

Enter table start value

Edit Plot→

X	F1		
1.5	3.5		
1.6	3.6		
1.7	3.7		
1.8	3.8		
1.9	3.9		
2	undefined		
2.1	4.1		
2.2	4.2		
2.3	4.3		
2.4	4.4		
2.5	4.5		

2

Zoom Size Defn Column

X	F1		
1.99995	3.99995		
1.99996	3.99996		
1.99997	3.99997		
1.99998	3.99998		
1.99999	3.99999		
2	undefined		
2.00001	4.00001		
2.00002	4.00002		
2.00003	4.00003		
2.00004	4.00004		
2.00005	4.00005		

2

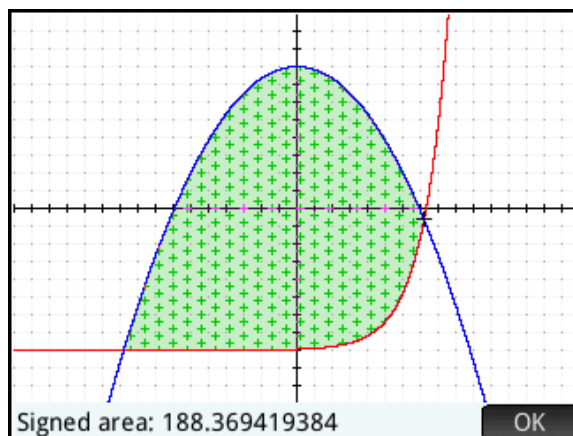
Zoom Size Defn Column

Example: Dividing Land Equally

Two brothers inherit land that they want to divide equally between them. The local land records indicate that the land is bounded roughly by the two functions $y = 8 - \frac{x^2}{6}$ and $y = \frac{2^{x-1}}{10} - 8$, where x and y are measured in kilometers. If they choose to put up a fence along a border running north and south, where should they put the fence?

1. Enter the functions in F1(X) and F2(X)
2. Use Plot or Numeric view to establish that the two boundaries intersect at $x \approx -9.8$ and $x \approx 7.2$
3. Use Signed Area to estimate the area of the land to be 188.37 km^2 (18,837 hectares), so each brother should get approximately 94.185 km^2
4. Optional: use Signed Area to get a visual estimation of the boundary
5. Define F3(X) as follows:

$$F3(X) = \int_{-9.8}^x F1(T) - F2(T) \delta T$$
6. Use Plot view to get an estimate ($x \approx -0.7$) and then use Numeric View to zoom in on the solution: $x \approx -0.644$



X	F1	F2	F3
-1.3	7.718333333	-7.97969369	8.3804396E1
-1.2	7.76	-7.97823624	8.5376238E1
-1.1	7.798333333	-7.97667418	8.6951929E1
-1	7.833333333	-7.975	8.8531125E1
-0.9	7.865	-7.97320566	9.0113480E1
-0.8	7.893333333	-7.97128254	9.1698650E1
-0.7	7.918333333	-7.96922139	9.3286288E1
-0.6	7.94	-7.96701230	9.4876045E1
-0.5	7.958333333	-7.96464466	9.6467574E1
-0.4	7.973333333	-7.96210709	9.8060524E1
-0.3	7.985	-7.95938738	9.9654545E1
-0.7			
Zoom		Size	Defn Column

X	F1	F2	F3
-0.65	7.929583333	-7.96813598	9.4080923E1
-0.649	7.929799833	-7.96811389	9.4096821E1
-0.648	7.930016	-7.96809178	9.4112719E1
-0.647	7.930231833	-7.96806966	9.4128617E1
-0.646	7.930447333	-7.96804752	9.4144516E1
-0.645	7.9306625	-7.96802536	9.4160414E1
-0.644	7.930877333	-7.96800319	9.4176313E1
-0.643	7.931091833	-7.96798100	9.4192212E1
-0.642	7.931306	-7.96795880	9.4208111E1
-0.641	7.931519833	-7.96793658	9.4224011E1
-0.64	7.931733333	-7.96791435	9.4239910E1
-0.644			
Zoom		Size	Defn Column

Extensions:

- What if the brothers wanted an east-west boundary? Estimate where that boundary should be.
- The brothers want to divide the land into thirds; where do the north-south boundaries occur?

In this example, we used the Function app to easily visualize and estimate the solution to a typical classroom problem. In our next example (P. 11), we extend that power of visualization.

The Advanced Graphing App

The Advanced Graphing App is designed to plot graphs in the x/y plane. It can handle conic sections, polynomials, inequalities – virtually any mathematical open sentence in x or y or both – or neither.

1. Press **Apps** and tap on the Advanced Graphing icon

The app opens in Symbolic view. There are 10 fields (S1-S9 and S0) for defining the graphs you want plotted in the Plot view.

2. In S1, enter $X^2 + 3Y^2 + 2XY - 81 = 0$
3. Tap on the color picker to choose a color for the graph
4. Press **Plot** to see the graph of S1
5. Tap **Menu** to open the menu

The menu is basically the same as the menu in the Plot view of the Function app, though without the **Fcn** menu.

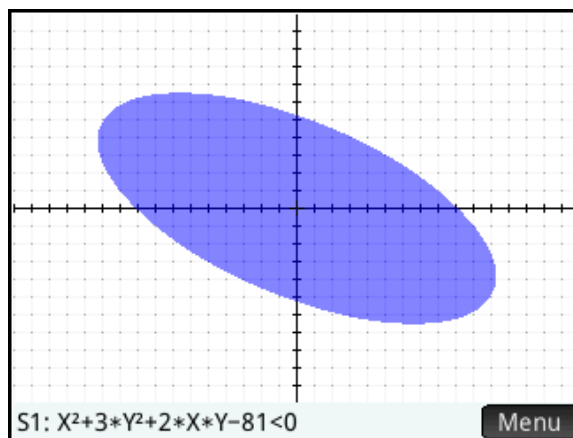
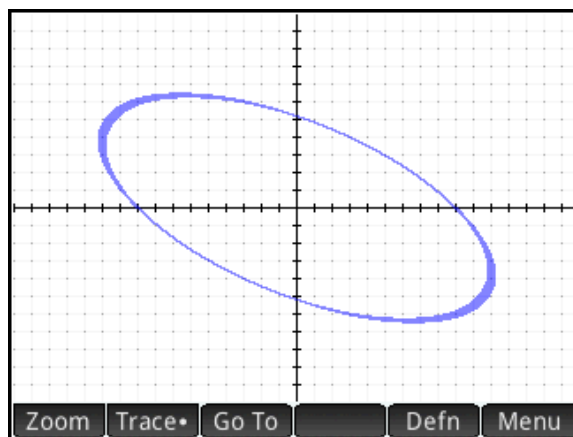
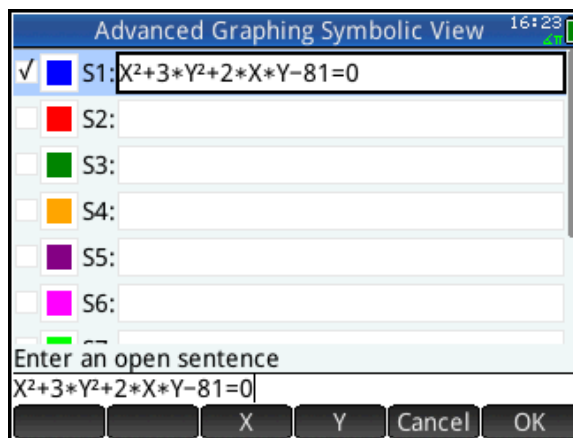
Things you can do:

- Tap anywhere on the display to re-locate the cursor
- Press **+** and **-** to zoom in and out on the cursor
- Tap and drag to scroll the graphing window
- Tap **Defn** to edit the current relation

6. Tap **Defn** and an editor opens, showing you the current expression in textbook format. Tap **Edit** and change the $=$ to $<$.

Hint: press **Shift** **6** to open a menu of common relational operators and tap $<$.

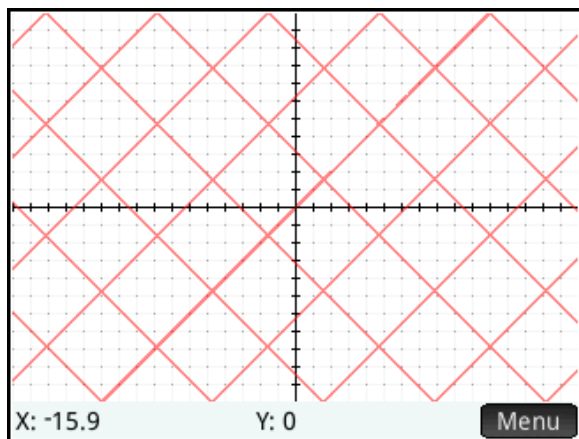
7. Tap **OK** to see the graph of the inequality and tap **↓** to exit the editor



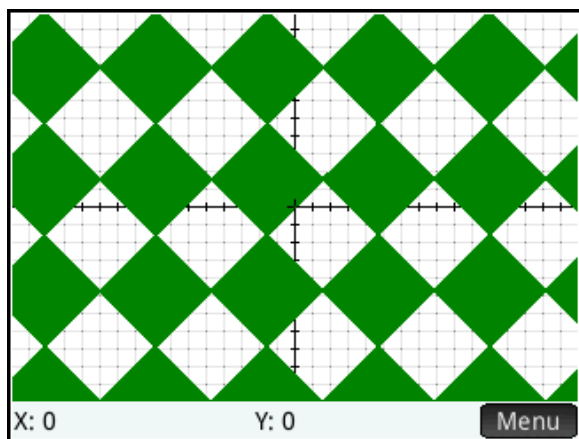
The Advanced Graphing App can plot the graphs of many types of relations. The table below lists just a few.

Relations	Examples	Notes
Polynomials in x and y	$x^2 + 3x^2 + 2xy - 81 = 0$	a rotated ellipse
	$4y^4 - 5x^2y^2 + x^4 = 0$	check out the factors
Linear Inequalities	$2x + 3y < 5$	
Non-Linear Inequalities	$1 > 0$	plots every pixel
	$\frac{ x-3 }{2} < 4$	
	$\text{Sin}\left(\left(\sqrt{x^2 + y^2} - 5\right)^2\right) > \text{Sin}\left(8 * \text{Tan}^{-1}\left(\frac{y}{x}\right)\right)$	see below

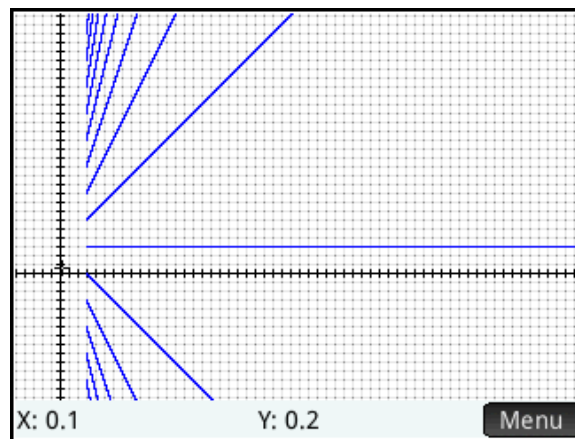
The gallery below shows some example graphs.



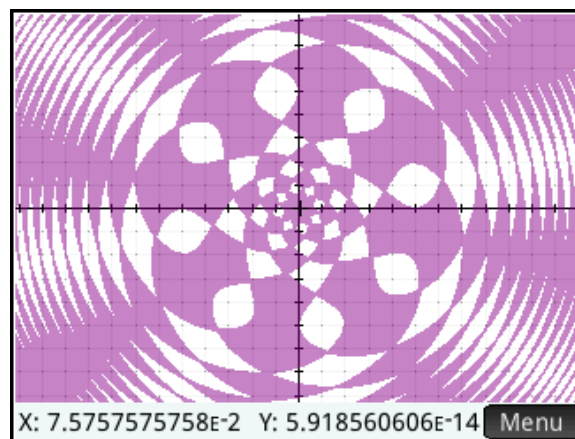
$\text{Sin}(x) = \text{Sin}(y)$



$\text{Sin}(x) < \text{Sin}(y)$



$y \text{ Mod } x = 3$



$\text{Sin}\left(\left(\sqrt{x^2 + y^2} - 5\right)^2\right) > \text{Sin}\left(8 * \text{Tan}^{-1}\left(\frac{y}{x}\right)\right)$

Example: Implicit Differentiation

If $4y^4 - 5x^2y^2 + x^4 = 0$, find $\frac{\partial y}{\partial x}$. This is a typical implicit differentiation problem. The solution

shows that the derivative depends on the values of both x and y , but how do students understand this? In this example, we extend the power of visualization to problems of this sort.

- Though it will not perform implicit differentiation directly, the CAS does handle it in steps, as shown to the right

Hint: enter the expression first, to keep a copy handy. Use Copy to insert it in your subsequent work.

Now we turn to the Advanced Graphing App to explore further.

- Press **Apps Info** and tap on the Advanced Graphing icon

- Enter the equation in S1

- Press **Plot Setup** to see the graph

The graph appears to be the lines, $y=x$, $y=-x$, $y=x/2$, and $y=-x/2$. The CAS factor command gives us a way to verify this.

- Press **CAS Settings** to return to the CAS view.
- Press **Mem B**, tap 'Algebra, and select ⁴factor. Then tap on the history and drag back up to your original expression. Tap on it to select it and then tap **Copy**.

Press **Enter** to see the factors of our expression.

The factors agree with our understanding of the graph. But if the graph consists of those four lines, then the derivative is limited to the values -1, -1/2, 1/2, and 1. How do we reconcile this with the rational expression we have for our derivative?

CAS Geometry 14:07

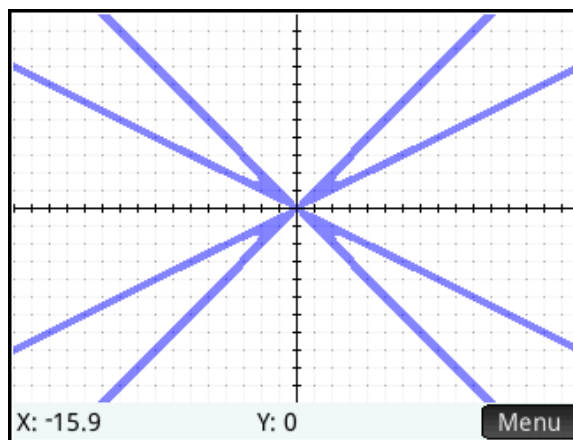
$$\frac{\partial}{\partial x} (4y^4 - 5x^2y^2 + x^4) = -10xy^2 + 4x^3$$

$$\frac{\partial}{\partial y} (4y^4 - 5x^2y^2 + x^4) = 16y^3 - 5x^2y$$

simplify $\left(\frac{-10xy^2 + 4x^3}{16y^3 - 5x^2y} \right)$

$$\frac{2x^3 - 5x^2y}{5xy^2 - 8y^3}$$

Sto ► simplif



CAS Geometry 14:09

$$\frac{\partial}{\partial y} (4y^4 - 5x^2y^2 + x^4) = 16y^3 - 5x^2y$$

simplify $\left(\frac{-10xy^2 + 4x^3}{16y^3 - 5x^2y} \right)$

factor $(4y^4 - 5x^2y^2 + x^4)$

$$(y-x)(y+x)(2y-x)(2y+x)$$

Sto ► simplif

7. Return to CAS view (**CAS**) and tap **simplif**. Press $\frac{\square}{\square}$, and select $\square \square$ (the Where command). This command is used to make substitutions.
8. Tap and drag the history until you see our derivative expression. Select it and tap **Copy**.
9. Move to the second box in the template and enter $y=x$

Hint: use lowercase variables in the CAS

10. Press **Enter** to see the result.
11. Simply copy the previous input and edit it for the other substitutions ($y = x/2$, etc.)

CAS Geometry 14:12

simplify $\frac{1-10xy+4x^2}{3}$ $\frac{2x-5xy}{2}$ $\frac{3}{5x-y-8y}$

factor $\left(\begin{array}{c} 4x^2-5xy+y^2 \\ 4xy-5x^2+y^2+x \end{array} \right)$ $(y-x)(y+x)(2y-x)(2y+x)$

simplify $\frac{3}{2x-5xy}$ $\frac{2}{3}$ $\frac{5x-y-8y}{y=x}$ 1

Sto ► simplif

CAS Geometry 14:13

simplify $\frac{3}{2x-5xy}$ $\frac{2}{3}$ $\frac{5x-y-8y}{y=x}$ 1

simplify $\frac{3}{2x-5xy}$ $\frac{2}{3}$ $\frac{5x-y-8y}{y=\frac{x}{2}}$ $\frac{1}{2}$

Sto ► simplif

In this example, we extended the power of visualization to include a polynomial in x and y . The graph of the polynomial led to a better understanding of its derivative and to a conjecture regarding its properties, and the power of the CAS let us prove the conjecture. HP Prime, with its CAS and the Advanced Graphing app, represents a new standard in classroom tools to explore mathematics, make conjectures, and pursue proofs.

The Geometry App

The Geometry app delivers the power of a dynamic geometry app in the palm of your hand. And its CAS integration gives you an analytic geometry app so you can prove your conjectures.

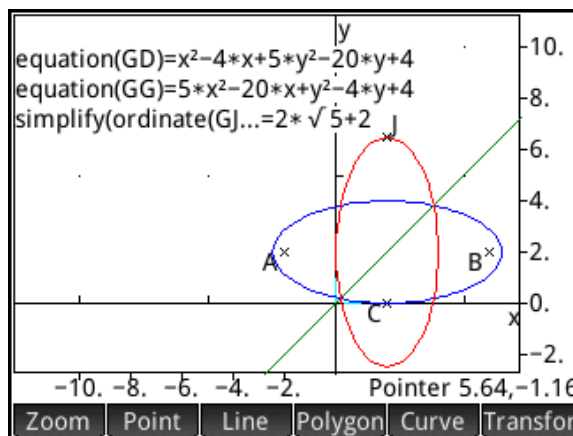
Press **Apps** and tap on the Geometry icon.

The app opens in Plot view. The zoom button (**Zoom**) opens the familiar Zoom menu with the same options as the other HP apps. Each of the other menu buttons opens menus to create various geometric objects:

- **Zoom:** zoom in or out, etc.
- **Point:** points, points on objects, midpoints, intersections, etc.
- **Line:** segments, lines, tangents, perpendiculars, etc.
- **Polygon:** triangles, quadrilaterals, and special polygons
- **Curve:** circles and other conic sections, locus of points, graphs of functions, etc.
- **Transform(m):** translation, reflection, dilation, etc.

Things you can do:

- Tap and drag to scroll the graphing window
- Press **Ans** and **Base** to zoom in and out on the cursor
- Tap any point and press **Enter** to select it
- Drag any selected point with your finger to move it
- Press **Symb** to view, edit, and create symbolic definitions of the objects in Plot view (and to hide or show them)
- Press **Num** to view, edit, and create measurements and calculations based on the objects in Symbolic view (and to show or hide them in Plot view)



Geometry Symbolic View 12:01

```

✓ GA:=point(-2,2)
✓ GB:=point(6,2)
✓ GC:=point(2,0)
✓ GD:=ellipse(GA,GB,GC,('display') = blue)
✓ GE:=line(y = x,('display') = green)
✓ GG:=reflection(GE,GD,('display') = red)

```

Bottom menu bar: Edit, ✓, New, ↑, ↓, Delete

Geometry Numeric View 11:59

```

✓ equation(GD) : x^2-0.0569822556861*x+y-0.28
✓ equation(GG) : x^2-0.0219585275121*x*y-4.09

```

Bottom menu bar: Edit, ✓, New, ↓, Delete

Example 1: Exploring Quadrilaterals

In this activity, we use the Geometry app to create a quadrilateral. We then create and connect the midpoints of consecutive sides of the quadrilateral to form another quadrilateral and explore the properties of the latter in terms of the former.

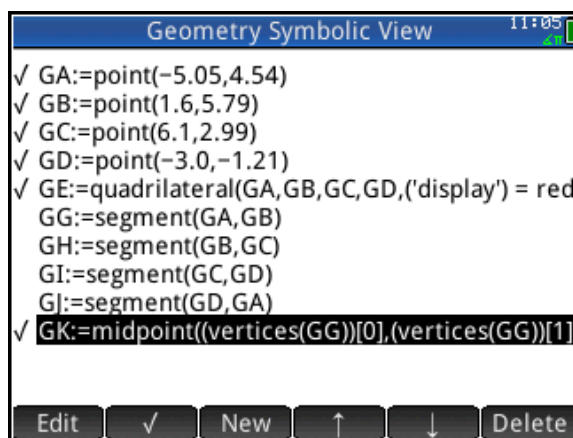
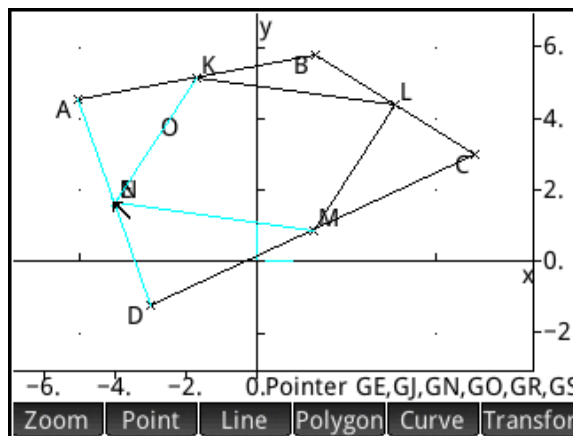
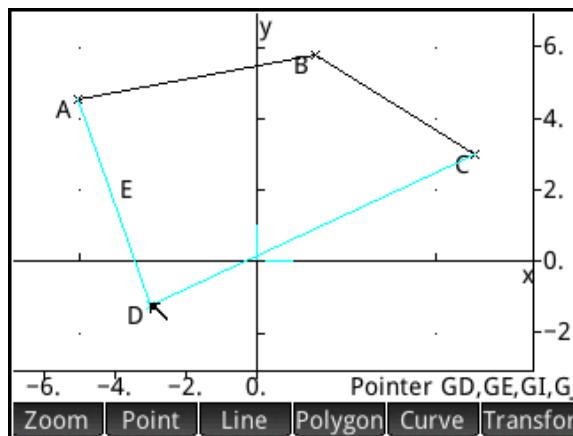
1. Press **Apps** and tap the Geometry icon.
The app opens in its Plot view.
2. Tap **Polygon** and select ²Quadrilateral.
3. Tap a location and press **Enter** to select the first vertex of the quadrilateral. Continue to tap and press **Enter** to select the other three vertices. Press **Esc** to deselect the quadrilateral command.








The display now shows a quadrilateral named E, based on the four points A, B, C, and D.

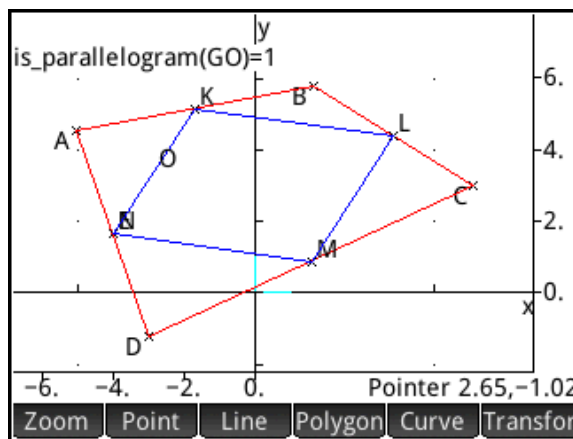
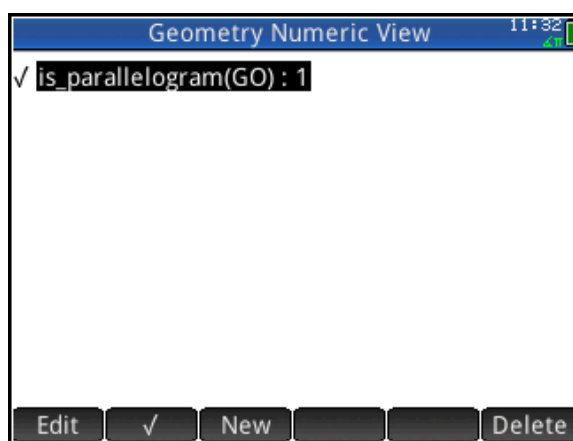
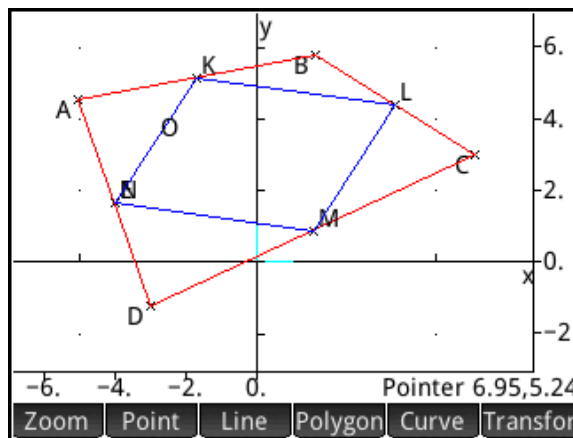
4. Tap **Point** and select ³Midpoint. Tap near the midpoint of \overline{AB} and press **Enter**. Continue likewise to create the midpoints of the other segments. When you are done, press **Esc** to deselect the midpoint command.
5. Repeat Steps 2 and 3 to create quadrilateral O from points K, L, M, and N.

The display now shows both quadrilaterals. We will now tidy up our construction and making it colorful before beginning our explorations.

6. Press **Symb** to display the Symbolic view of the app. Here, each of the geometric objects is defined symbolically. Objects with checks besides them are displayed in the Plot view. Uncheck each segment by highlighting it and pressing **✓**. Scroll down to uncheck the segments KL, LM, MN, and NK as well.



7. With the segments hidden, return to the Plot view (press ) and give the quadrilaterals their own colors. Press  to open the context-sensitive menu and select ³Change Color. A choose box pops up with the various geometric objects. Tap on GE:=quadrilateral(GA,...; then tap on the red box in the color picker.
8. Repeat Step 7 to select dark blue for the inner quadrilateral. You will have to drag with your finger to scroll down the choose box of objects to locate GO:=quadrilateral(GK,...
9. Press  to open the Numeric view of the app. Here we define measurements and tests involving our geometric objects. Tap  to start a new measurement. In this case, it is a test. Tap  to open the menu of commands select ²Tests, then select ^Ais_parallelogram. The command is pasted into the command line. Remember that the name of our inner quadrilateral is GO. Type "GO" between the parentheses and press . Tap  to check this test for display in the Plot view.
10. Return to the Plot view to see our constructions and the test result. Drag anywhere outside of the construction to pan the plane so you can see both the test result and the construction.



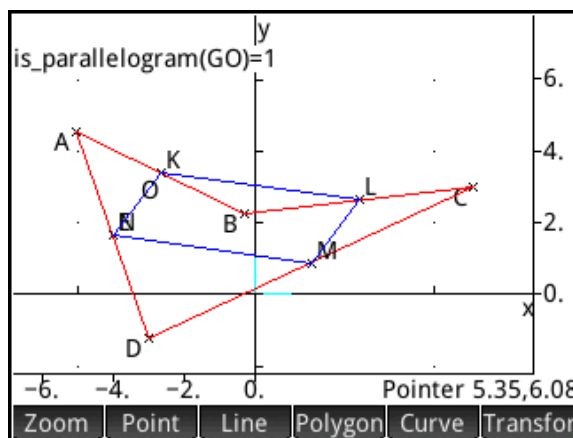
We are now ready to explore our construction.

1. Select one of the vertices of the outer quadrilateral by tapping on it and pressing **Enter**. You can now drag it anywhere within the display with your finger. As you move a vertex, notice that the parallelogram test on KLMN maintains a value of 1, indicating it is always a parallelogram. The parallelogram test can return any of 5 values:

0. Not a parallelogram
1. A parallelogram only
2. A rhombus
3. A rectangle
4. A square

You can now explore by dragging, but there is also another way to explore the effects of the properties of ABCD on KLMN.

2. Press **Symb** to return to the Symbolic view. Here you can give the coordinates of points A, B, C, and D exact values.
3. Select GA, tap **Edit** and enter new coordinates (-3,3). Tap **OK** when you are done.
4. Repeat Step 3 with points B, C, and D so that you have A(-3,3), B(3,3), C(3,-3) and D(-3,-3), making ABCD a square.
5. Press **Plot** to return to the Plot view. The display shows the parallelogram test now has a value of 4, indicating that KLMN is a square as well.



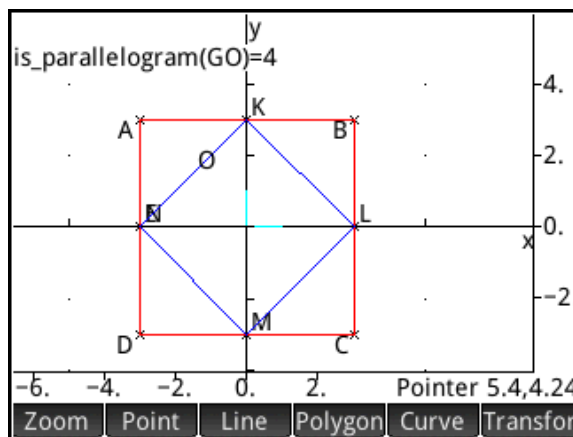
Geometry Symbolic View 11:47

```

✓ GA:=point(-3,3)
✓ GB:=point(3,3)
✓ GC:=point(3,-3)
✓ GD:=point(-3,-3)
✓ GE:=quadrilateral(GA,GB,GC,GD,('display') = red)
GG:=segment(GA,GB)
GH:=segment(GB,GC)
GI:=segment(GC,GD)
GJ:=segment(GD,GA)
✓ GK:=midpoint((vertices(GG))[0],(vertices(GG))[1])

```

Edit ✓ New ↑ ↓ Delete



It seems that KLMN is always at least a parallelogram, no matter where we move the coordinates of points A, B, C, and D (as long as they are not collinear!). To see why this is so, simply construct the diagonals AC and BD.

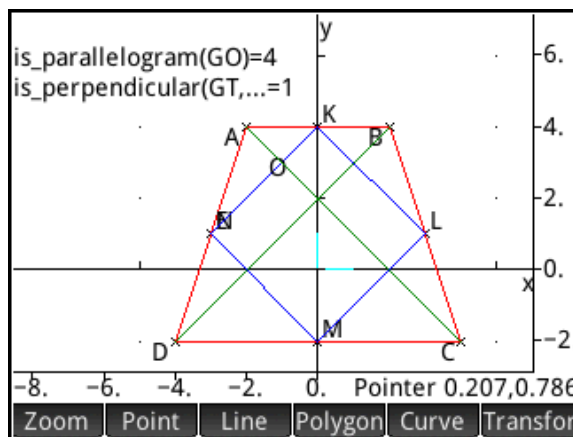
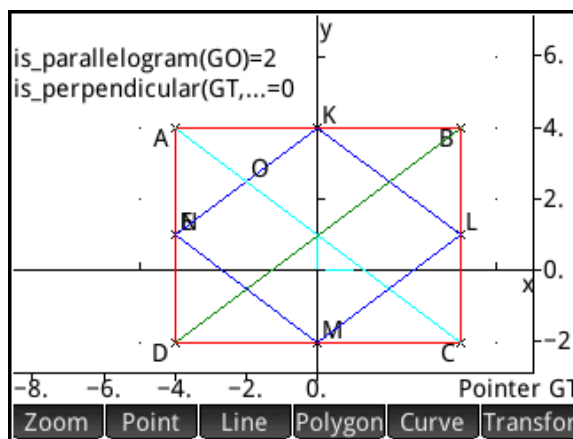
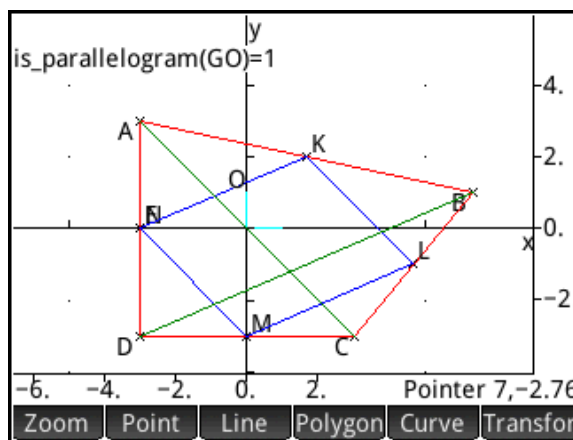
6. Tap **Line** and select ¹Segment. Tap on point A and press **Enter**. Then tap on Point C and press **Enter**. Repeat with points B and D. Then press **Esc** to deselect the command.

Since segment KL joins the midpoints of two sides of $\triangle ABC$, then it is parallel to the third side (AC); likewise, MN joins the midpoints of two sides of $\triangle ACD$, and is thus parallel to AC as well. Thus KL and MN are parallel. The same can be seen for segments NK and ML.

7. Press **Apps** and tap **Save** to save your construction with a name you'll remember

With the testing and symbolic abilities of the Geometry app, students can set about the proofs or refutations of conjectures such as these:

- If ABCD is a rhombus, then KLMN is a rectangle
- If ABCD is a rectangle, then KLMN is a rhombus
- If ABCD is an isosceles trapezoid, then KLMN is a rhombus
- If the diagonals of ABCD are perpendicular, then KLMN is a rectangle
- If the diagonals of ABCD are congruent, then KLMN is a rhombus



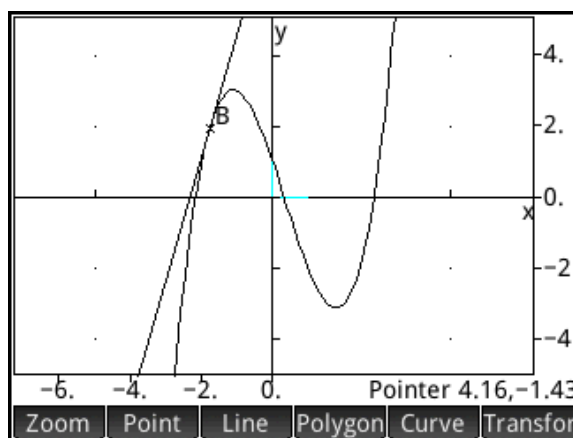
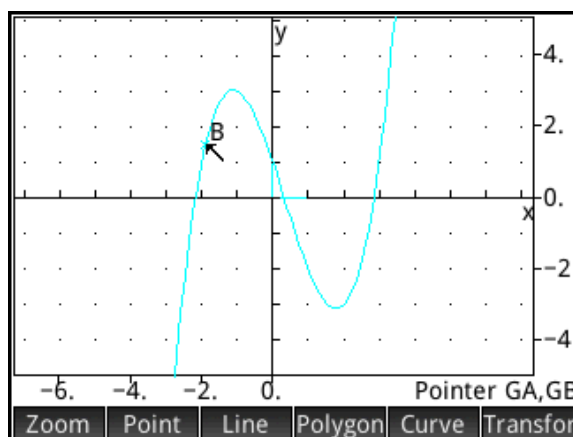
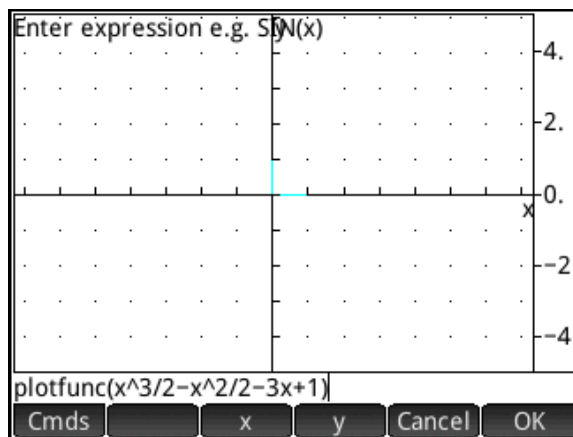
Example II: Slope and Derivative of a Function





In this activity, we construct a visualization tool for the derivative of a function.

1. With your previous construction saved, press **Apps Info**, use the direction keys to select the Geometry icon, and tap **Reset**. You will be asked to confirm the reset; tap **OK**.
2. Tap **Curve**, tap ⁶Plot, and select ¹Function. If there are any functions defined in the Function app, a window pops up asking if you want to use any of them or create a new one. Select New.
3. An editor opens with `plotfunc(` and new menu buttons appear with `x` and `y`. Enter the function $\frac{x^3}{2} - \frac{x^2}{2} - 3x + 1$ after the parenthesis and tap **OK**. The graph of the function is drawn. Press **Ans** to zoom in.








Note: the Geometry app uses lowercase `x` as the independent variable for functions.

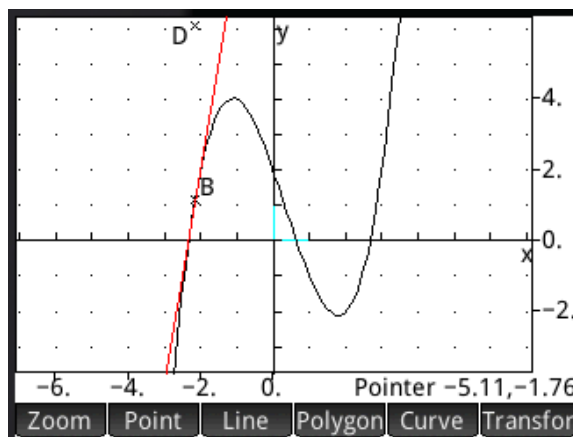
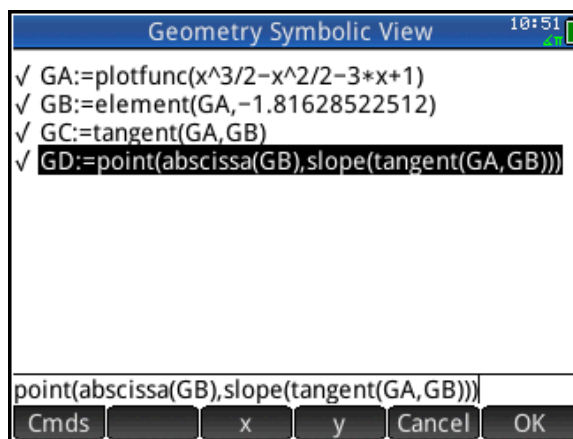
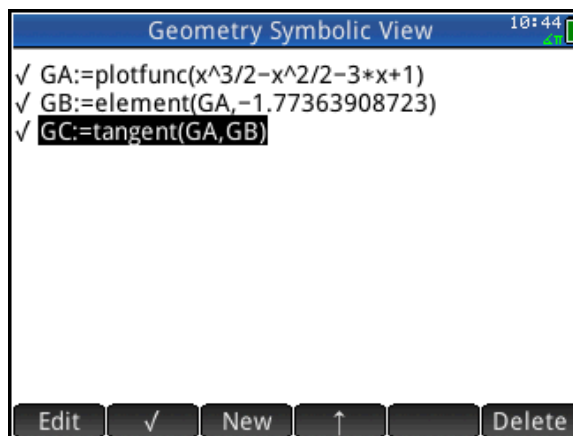
4. Tap **Point** and select ²Point On. Tap on the graph (it will turn blue when selected) and press **Enter**. Point B appears, defined as a point on the graph. Press **Esc** to exit this command.
5. Tap **Line**, tap ⁵More, and select ⁵Tangent. Tap on the curve and press **Enter**. Then tap on point B. A message at the bottom should read `tangent(GA,GB)`. Press **Enter**. The tangent to the function through point B is drawn.



6. Tap point B and press  to select it. You can now drag point B along the curve with your finger or tap on a new location and the tangent line will follow.
7. Press  to see the Symbolic view of the app. All objects created in the Plot view have their symbolic definitions here.
8. Tap  to create new object. Tap , tap ¹Point, and select ⁵Point. The point() command appears in the edit line.

We will now define the coordinates of this point. The x-coordinate will be the same as that of point B and the y-coordinate will be the same as the slope of the tangent. The new point D will be a point on the derivative of the function.

9. Press  and tap  to open the Catalog. Press   to jump to commands that start with A and find abscissa. Tap . Between the parentheses, type GB. Move past the right parenthesis, press  and return to the Catalog for the slope and tangent functions. The completed definition is shown to the right.
10. Press  to return to Plot view. As you move point B, you can see that point D moves along the derivative.



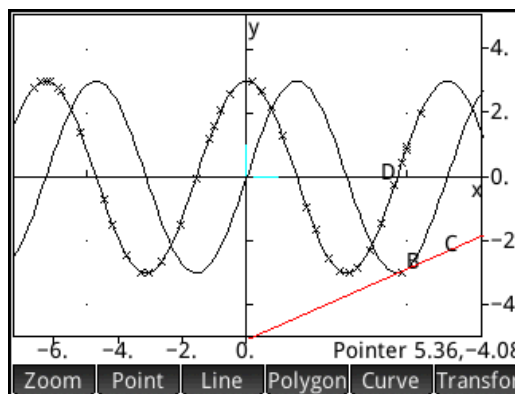
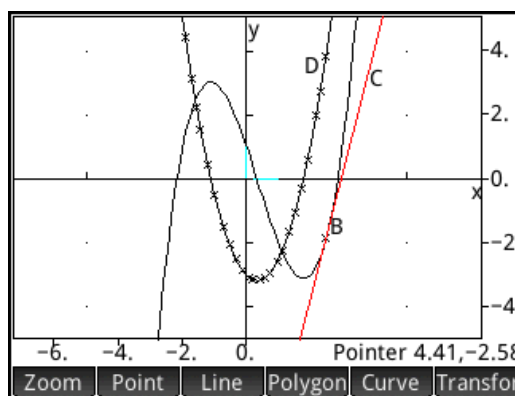
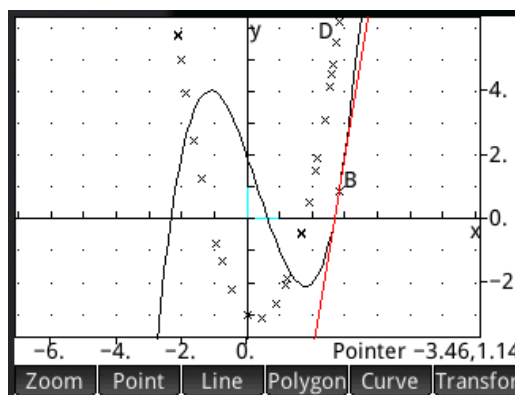
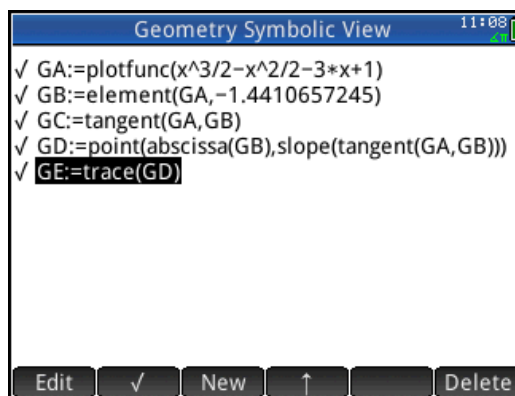
We are now ready to use our construction!

1. Return to Symbolic view and create a new object, GE. As shown to the right, $GE:=\text{trace}(GD)$.
2. Return to Plot view, select point B and move it. Each time you tap the display to move point B, a small marker will be left to show where point D was. As you continue to tap, the shape of the derivative function emerges, as shown to the right.
3. Uncheck GE in Symbolic view to erase the trace and keep it from drawing in Plot view
4. Return to Symbolic view and create GF. Define $GF:=\text{locus}(GD,GB)$, which means the locus of point D as point B moves along the graph. This new object represents the derivative function, as shown right and below. Check and uncheck it in Symbolic view to show and hide it in Plot view.

You now have a construction that has a number of methods to visualize the derivative of a function:

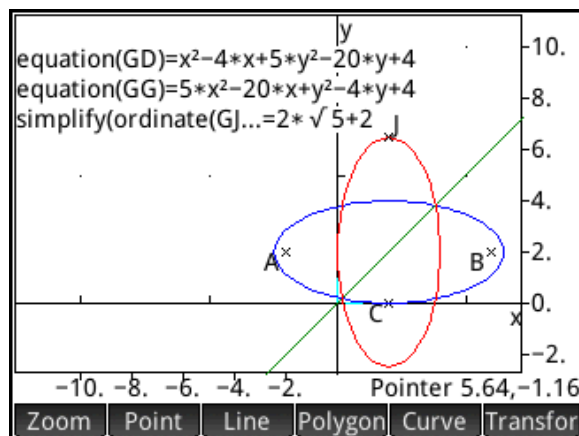
- Move point B and see a single point on the derivative that has the same x-value as point B
- Move point B and leave a trace of points on the derivative
- View the entire locus of points on the derivative

As you can see from the figure to the right, you can return to Symbolic view and change GA to be the graph of any function. The trace and locus will continue to work. Your students can now visualize the graph of the derivative of any function using this one construction. Save your construction now with a name you will remember!

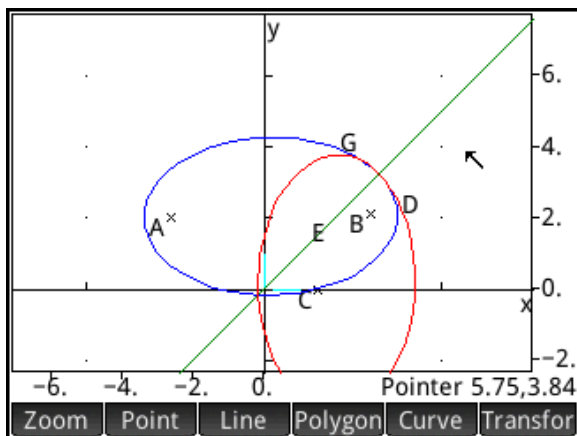
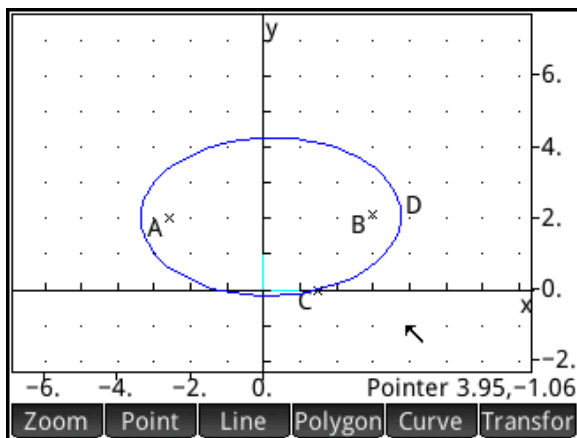


Example III: Reflection and Inverse

In this construction, we have a blue ellipse defined by the focal points A and B, and a point C on the ellipse. The red ellipse is the reflection of the blue one over the line $y=x$, so they are inverses of each other. Note the equations of the two ellipses and how the roles of x and y are interchanged. Since the points A, B, and C were defined exactly in the Symbolic view, the equations are exact as well. Also, the y -coordinate of point J is shown to emphasize the exact nature of the result.



1. With your previous construction saved, reset the Geometry app
2. Tap **Curve** and select ²Ellipse. Tap at the location of the first focus and press **Enter**. Then proceed with the second focus and a point on the ellipse. Press **+** to zoom in and drag to scroll the window. Change the color of your ellipse to blue.
3. In Symbolic view, create $GE:=\text{line}(y=x)$
4. Back in Plot view, tap **Transform** and select ²Reflection. Tap the line and press **Enter**. Then tap the ellipse and press **Enter**. Each time you select an object, it should turn light blue to indicate it is selected. Press **Esc** to exit the reflection command. Change the color of the reflected ellipse to red.



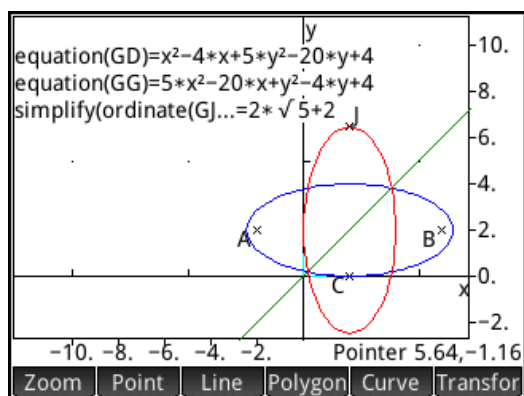
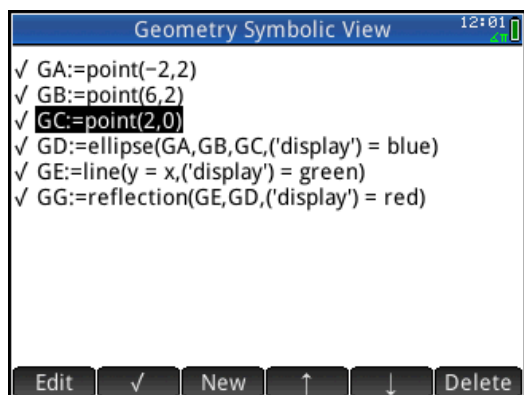
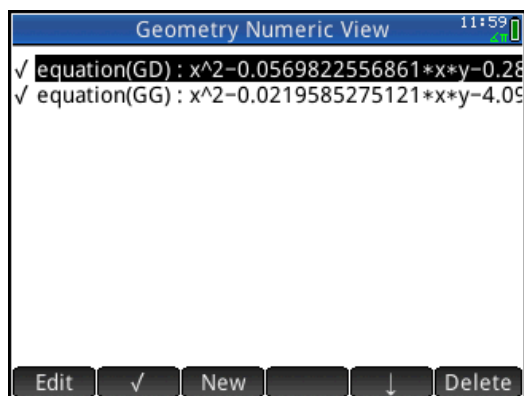
The Numeric view of the Geometry app is where you go to make measurements or create calculations based on measurements. Check the ones you want displayed in the Plot view.

5. Press **Num** to see Numeric view. Tap **New** to create a new measurement or calculation. Tap **Cmnds** to open the Commands menu, tap ¹Measure, and select ²equation. After the parentheses, type GD (the name of the first ellipse) and tap **OK**.
6. Repeat Step 5 to get the equation of the reflected ellipse (GG)
7. Check both entries in Numeric view to ensure that they are visible in Plot view.

Note the approximate nature of the equations.

8. Return to Symbolic view and change the coordinates of points A, B, and C as shown to the right above.
9. Now when you return to Plot view, the equations are shown exactly.

You can see that the roles of x and y are interchanged in the two equations. Note also that the y-coordinate of Point J is reported as an exact value as well. The CAS can be used to retrieve the equation of the red ellipse and put it into a more familiar form. Note the length of the major semi-axis is $\sqrt{20} = 2\sqrt{5}$ and the center is at (2,2). So the point J on the reflected ellipse should have a y-coordinate of $2 + 2\sqrt{5}$, which it does! The integration of CAS and dynamic geometry gives you an exact analytical geometry app that is dynamic as well.



CAS Geometry 12:18

equation(GG)

$$5x^2 - 20x + y^2 - 4y + 4$$

collect

$$\frac{\text{factor}\left[5x^2 - 20x + 20\right]}{20} + \frac{\text{factor}\left[y^2 - 4y + 4\right]}{20} =$$

$$\frac{(x-2)^2}{4} + \frac{(y-2)^2}{20} = 1$$

Sto ► simplif

The Spreadsheet App

The Spreadsheet App gives you the most common features you expect in a spreadsheet. But with HP Prime, you also get the power of a CAS integrated with the spreadsheet.

Press **Apps** and tap the Spreadsheet icon. The app opens in Numeric view-its only view.

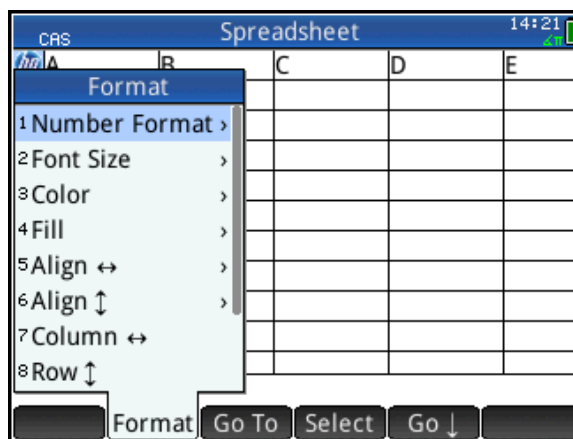
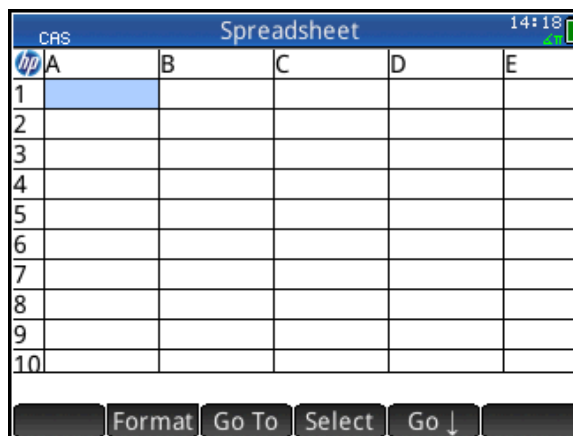
The menu keys are:

- **Format:** opens the Format menu (see figure to the right and below)
- **Go To:** jumps to a specified cell
- **Select:** activates Selection mode
- **Go:** determines which cell is selected when you tap **OK**
- **Show:** displays the cell contents in textbook format

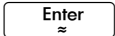
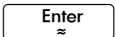

Things you can do:

- Tap and drag to scroll through the spreadsheet
- Tap and hold to invoke Selection mode; then drag to select a rectangular block of cells
- Pinch open or closed horizontally to change the width of a column
- Pinch open or closed vertically to change the height of a row
- Enter contents into a cell (number, matrix, expression)
- Define a cell, row, column or sheet using a formula

The Spreadsheet app can return numerical approximations for a formula, or it can use the CAS to return exact numeric or symbolic results. The examples in this section will cover both uses.







To get started in the Spreadsheet app, here is a simple exercise that illustrates the CAS and non-CAS uses of the app.






1. Select cell A1, type 12/15, and press .
2. Select cell A2 and type the same thing, but before pressing , tap . The dot next to CAS shows the CAS is now active



A1 is evaluated numerically, but A2 is simplified using the CAS. Select B1 to see the acronym CAS at the start of its formula.

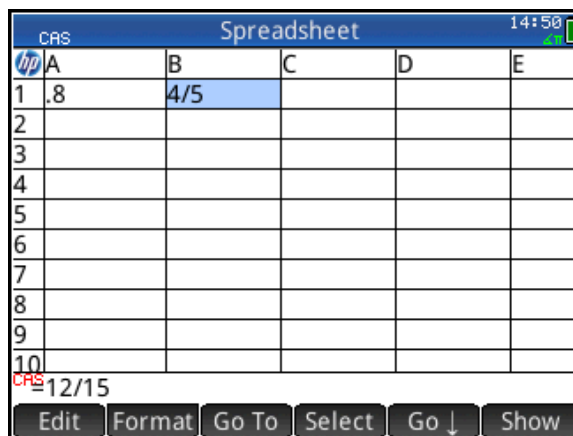
Example 1: Pascal's Triangle

Next we define the entire spreadsheet with a single command.

1. Tap the upper-left corner to select the entire sheet. Press   to start a new formula. Then press , tap , tap 5P Probability, and select 3C Combinations. Between the parentheses, enter Row-1,Col-1, as shown to the right.

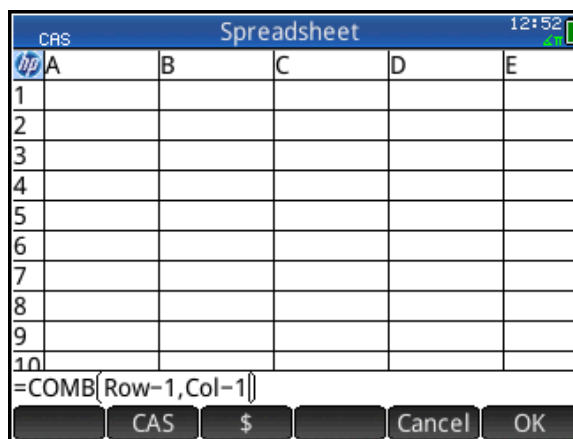
Hint: Row and Col are Spreadsheet app variables. To retrieve Row, press , tap , tap 1S Spreadsheet, tap 1N Numeric, and select 3R Row. Select Col in a similar manner. You can always just type names in letter by letter, using  for uppercase and  for lowercase letters.  for lowercase letters.

2. Tap  to see the spreadsheet fill with Pascal's triangle! Use your finger to scroll through the spreadsheet.
3. To clear the entire spreadsheet, tap on the upper-left corner and press .



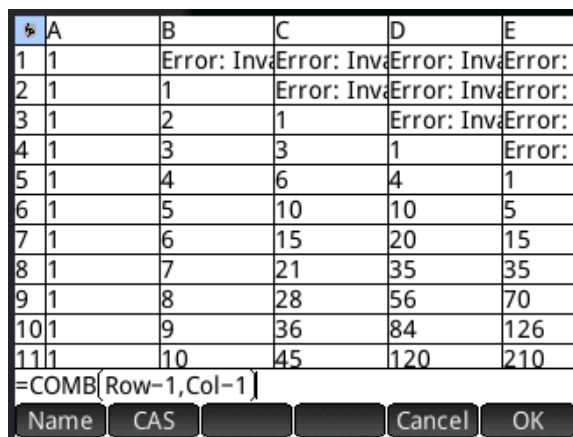
	A	B	C	D	E
1	12/15	4/5			
2					
3					
4					
5					
6					
7					
8					
9					
10					

Below the spreadsheet, the formula bar shows $=12/15$ and the CAS mode is active (indicated by a dot next to CAS).



	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Below the spreadsheet, the formula bar shows $=COMB(Row-1,Col-1)$ and the CAS mode is active.



	A	B	C	D	E
1	1	Error: Invalid	Error: Invalid	Error: Invalid	Error: Invalid
2	1	1	Error: Invalid	Error: Invalid	Error: Invalid
3	1	2	1	Error: Invalid	Error: Invalid
4	1	3	3	1	Error: Invalid
5	1	4	6	4	1
6	1	5	10	10	5
7	1	6	15	20	15
8	1	7	21	35	35
9	1	8	28	56	70
10	1	9	36	84	126
11	1	10	45	120	210

Below the spreadsheet, the formula bar shows $=COMB(Row-1,Col-1)$ and the CAS mode is active.

Example II: The Fibonacci Sequence

Another important variable for the Spreadsheet app is Cell, as you will see in our next example.

1. Tap on the column header for Column A to select it.
2. Press **Shift** $\frac{\square}{\square}$ to start a new formula. Then enter $\text{Cell}(\text{Row}-2,1)+\text{Cell}(\text{Row}-1,1)$ as shown to the right.
3. Tap **OK** to see Column A fill with zeroes.
4. But now enter 1 in cell A1 to see Column A fill with the Fibonacci sequence.

You can now appreciate the value of the app variables, Cell, Col, and Row!

Example III: Back to Pascal's Triangle

Here is another CAS example that involves symbolic results.

In the example to the right, we use the CAS to define one column of the spreadsheet to expand the binomial $x+1$ to various integer powers. Note the expression editor shows a CAS button: **CAS**. When active, the CAS is used to evaluate the formula. When it is not active, the formula is used to obtain numerical results.

Imagine all the patterns you and your students can explore in symbolic expressions!

	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Formula: $=\text{Cell}(\text{Row}-2,1)+\text{Cell}(\text{Row}-1,1)$

	A	B	C	D	E
1	1				
2	1				
3	2				
4	3				
5	5				
6	8				
7	13				
8	21				
9	34				
10	55				

	A	B
1	$x+1$	
2	x^2+2x+1	
3	x^3+3x^2+3x+1	
4	$x^4+4x^3+6x^2+4x+1$	

Formula: $=\text{expand}((x+1)^{\text{Row}})$

Examination Mode

HP Prime can be configured and locked for an examination. The machine will remain locked for a pre-set time period and secured with a password. LED lights at the top of the unit will flash to show that it is in examination mode.

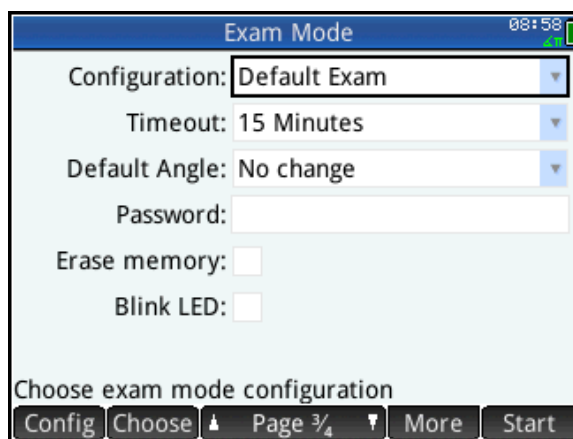
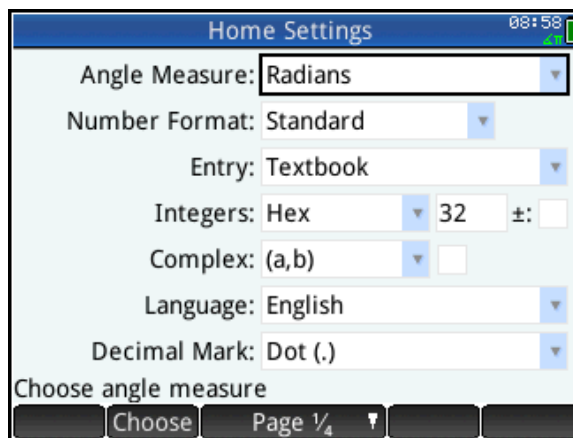
1. Press **Shift** **Settings** to enter Home Settings
2. Swipe upwards to get to the third page; the header will say *Exam Mode*.
Alternately, press the right halves of **Page 1/3** and **Page 2/3**.

The menu keys are:

- **Config**: opens the Configuration page, where you can check which features you want disabled
- **Choose**: opens a choose box
- **Page**: tap the left half to go up a page and the right half to go down a page in the Home Settings
- **More**: opens a menu of options to copy or reset the current configuration
- **Start/Send**: starts Exam mode on the current HP Prime or send it to another HP Prime (**Start** changes to **Send** if Prime is connected via USB)

Things you can do:

- Give your configuration a name
- Set a time period
- Set a password
- Check a box to erase memory when examination mode starts
- Check a box to make the LED lights blink while in examination mode.

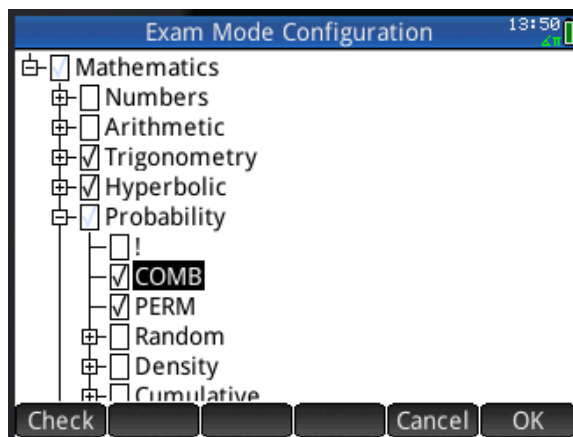
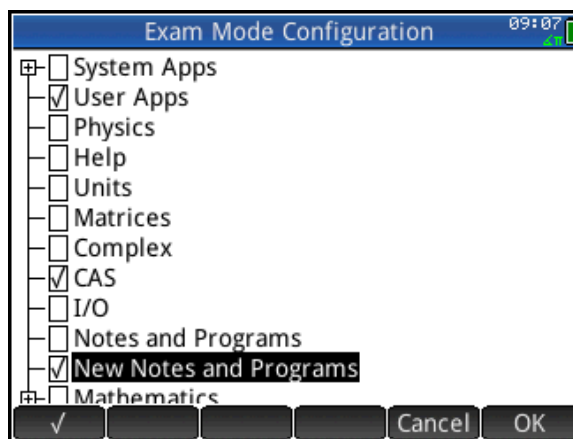
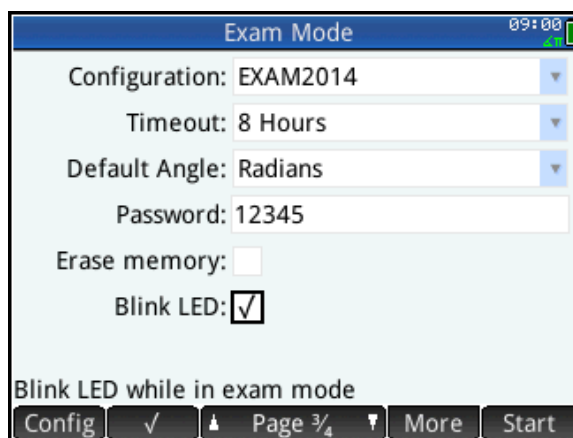


3. Tap *Configuration*, tap **More**, and enter a name for our new configuration: EXAM2014
4. Tap *Timeout*, then tap **Choose** and select a time period.
5. Tap *Default Angle*, then tap **Choose** and select a default angle measure
6. Tap *Blink LED* twice (to select it and check it)

We are now ready to define our EXAM2014 configuration.

7. Tap **Config** to enter the Configuration page
8. Tap *User Apps* to disable saved apps with their data
9. Tap *CAS* to disable the CAS
10. Tap *New Notes and Programs* to disable the Note and Program editors
11. Tap and drag to scroll down the tree
12. Tap on the plus sign (+) next to *Mathematics* to expand the tree
13. Tap on *Hyperbolic* to disable all hyperbolic trigonometric functions
14. Tap on the plus sign next to *Probability* to expand the tree another level and tap *COMB* and *PERM* to disable the individual functions ${}_nC_r$ and ${}_nP_r$
15. Tap **OK** to save this configuration with your new name: EXAM2014
16. Press **Start** to start Exam Mode on the device, or if it is connected to another HP Prime, press **Send** to start Exam Mode on the attached HP Prime

Once Exam Mode starts, the LED lights will blink to show that the configuration is in effect. All HP Prime calculators that were sent the same Exam Mode configuration from the same HP Prime will blink in unison using a random pattern of the 3 color lights.



The Data Streamer App

The Data Streamer App works with the HP StreamSmart 410 and up to four Fourier[®] sensors to collect data in real time. The final data set is sent to one of the statistics apps for analysis. Just plug 1-4 sensors into the StreamSmart 410 and start the Data Streamer App.

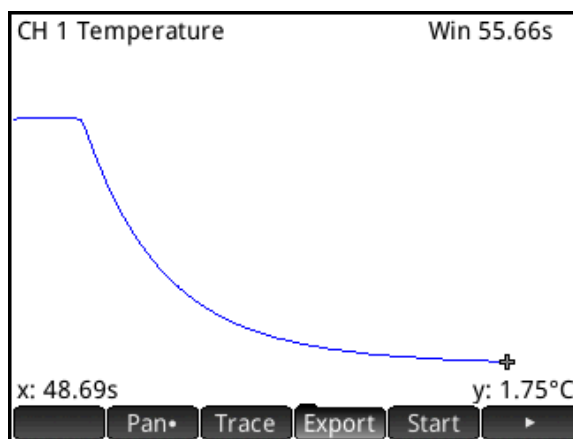
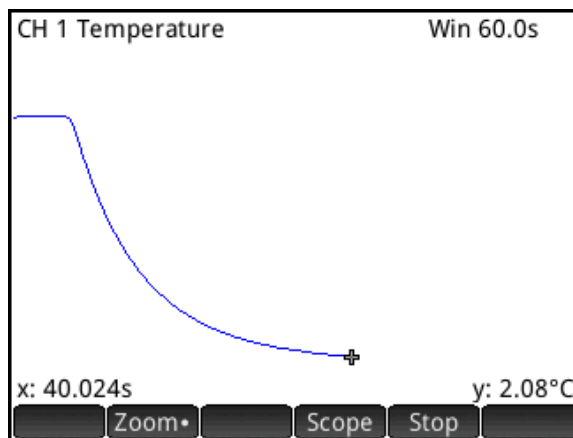
Press **Apps** and tap the DataStreamer icon.

The app opens in Plot view. The menu keys are:

- **Chan:** select one of the 4 channels for tracing (if more than one sensor is connected)
- **Pan/Zoom:** select whether the direction keys are used to Pan (scroll) or Zoom the Plot view
- **Trace:** toggles tracing off and on
- **Scope:** start oscilloscope mode
- **Export:** opens a menu to select data to export to a statistics app for analysis
- **Start/Stop:** starts and stops data streaming
- **▶:** displays a second page of options

Things you can do:

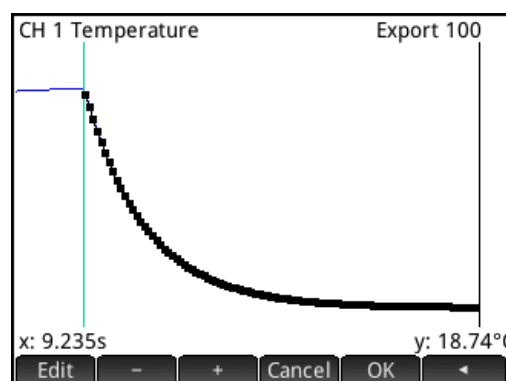
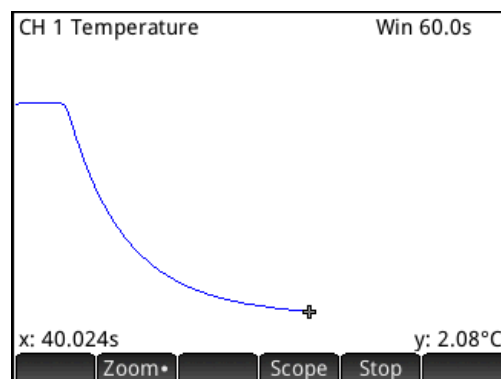
- With Pan active, press **▲** and **▼** to move the current data stream up and down in the Plot view
- Tap **Pan•** to change it to **Zoom**. Press **▶** and **◀** to zoom in or out on the Plot view while data is streaming. The effect is to speed up or slow down the data stream until it meets your needs.
- Tap **Stop** to stop data streaming. It changes to **Start**, ready for you to start another data stream
- Tap **Export** to open the Export menu, where you can select just the data you want and send it to one of the Statistics apps for analysis



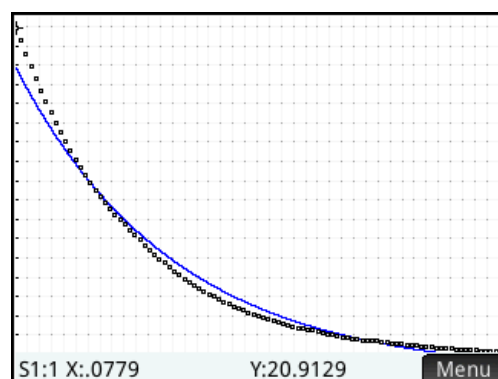
Example: Temperature Experiment

In this example, we plunge a temperature sensor into a beaker of ice water.

1. Attach a temperature sensor to the StreamSmart 410, and attach the StreamSmart 410 to your HP Prime via the USB cable
2. On the HP Prime, Press **Apps Info** and tap the DataStreamer icon
3. Tap **Start** to begin data streaming
4. While streaming and with Pan active (**Pan•**), press **▲** and **▼** to center the data stream in the display
5. Tap **Pan•** to change it to **Zoom** . Now press **▲** and **▼** to zoom in and out vertically so the stream fits in the display
6. When you see the data you want, tap to stop data streaming
7. Tap **Export** to open the Export menu.
8. Tap **I** and press **▶** and **◀** to crop data from the left
9. When you have selected the data you want, tap **▶** and **OK** . Tap **OK** again to start the Statistics 2Var app with your data.
10. Press **Symb** **↵** to enter the Symbolic view. Tap on *Type1*, tap **Choose** and select Exponential.
11. Press **View Copy** and select Autoscale to see the scatter plot of your data and the exponential fit.
12. Press **Symb** **↵** to return to the Symbolic view to see the equation of your fit

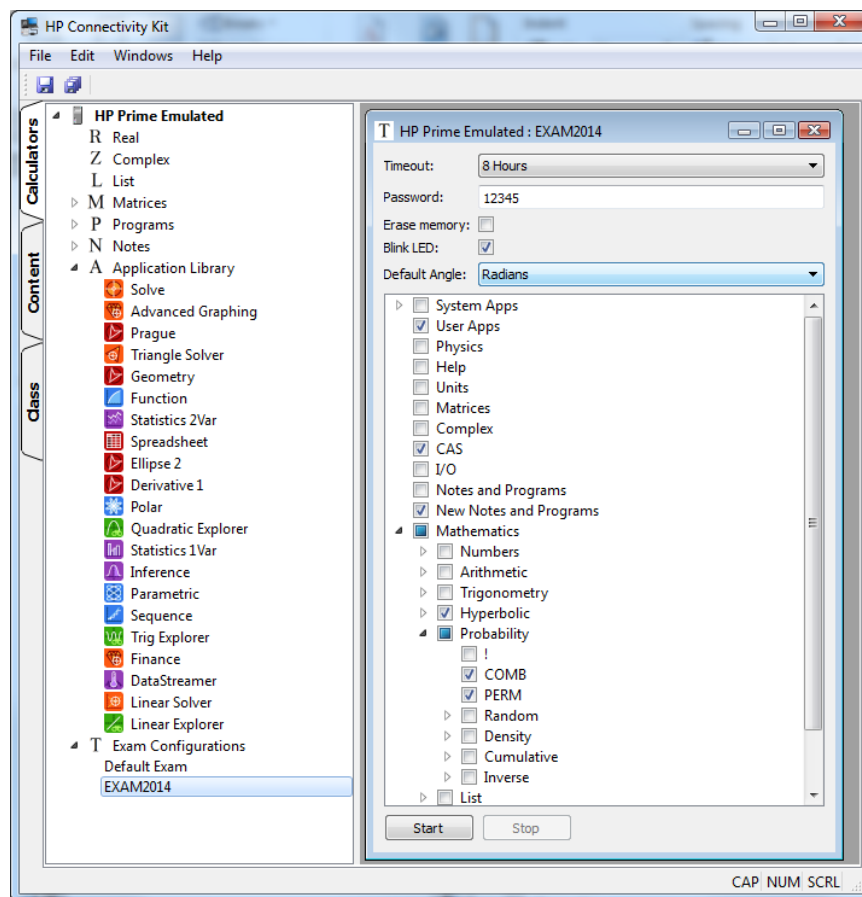


Statistics 2Var Numeric View 09:59				
	C1	C2	C3	C4
1	10.0368	17.5293		
2	10.5711	16.64		
3	11.1055	15.7507		
4	11.6398	14.9504		
5	12.1741	14.1796		
6	12.7105	13.4386		
7	13.2448	12.7864		
8	13.7791	12.1343		
9	14.3135	11.5414		
10	14.8478	11.0079		
10.0368				
<div> Edit Ins Sort Size Make Stats </div>				



The HP Prime Connectivity Kit

The HP Connectivity Kit allows you to connect to one or more HP Prime calculators via USB or wirelessly.



The Connectivity Kit has three panes:

- **Calculator Pane:** see the data on the connected calculator, edit apps, write programs, and synchronize the new data with the connected HP Prime calculator
- **Content Pane:** create and edit Exam Mode configurations, create polls and quizzes, etc.
- **Class pane:** see all HP Prime displays in your classroom network, monitor students, send apps, data, polls, quizzes, share one student's display for discussion purposes, send and start an Exam Mode configuration to the entire class, etc.

Appendix

Example: Local Linearity, the Difference Quotient and Limits

This activity uses the Function App in a split-screen view to explore the basic concept of local linearity and the limit of the difference quotient as underpinning the notion of derivative.

11. Press  and tap the Function icon.



The app opens in its Symbolic view.


12. Enter $F1(X) = \sin(X)$ and


$$F2(X) = \frac{F1(X) - F1(0)}{X - 0} \text{ as shown to the right.}$$

We use “X-0” in the denominator to reflect the difference quotient and to allow extensions.

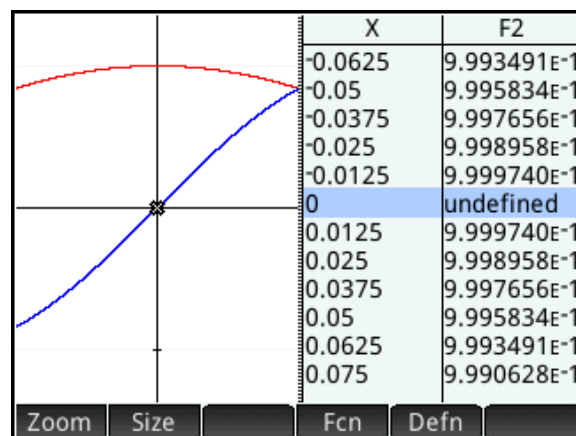
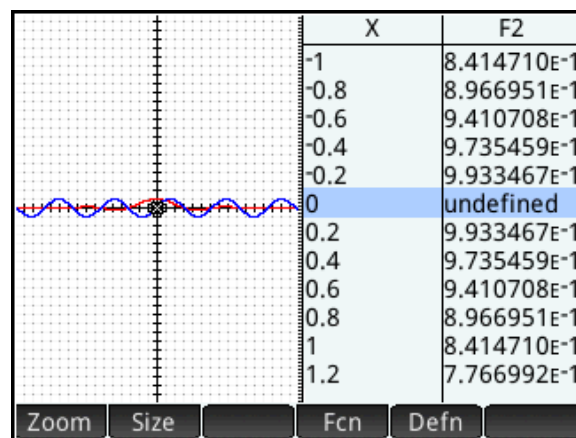
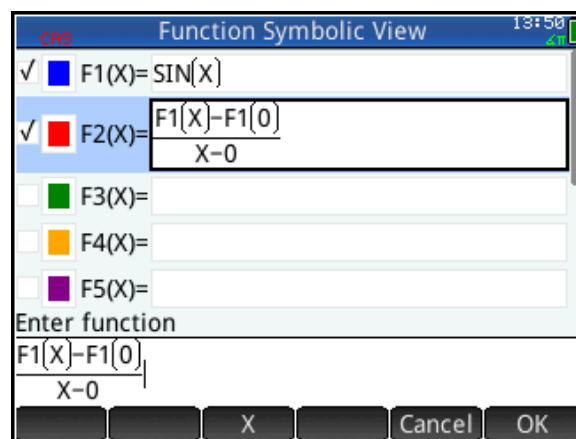
13. Press  and select Split Screen: Plot

Table. Press 0  to create a row in the table at $x=0$, then press  to trace $F2(X)$. The table shows the value of $F2(0)$ as undefined. The cursor appears to remain on $F1(0)$, since the value of $F2(0)$ is not defined.

14. Press  to zoom in on the graph of $F1(X)$ at $x=0$.

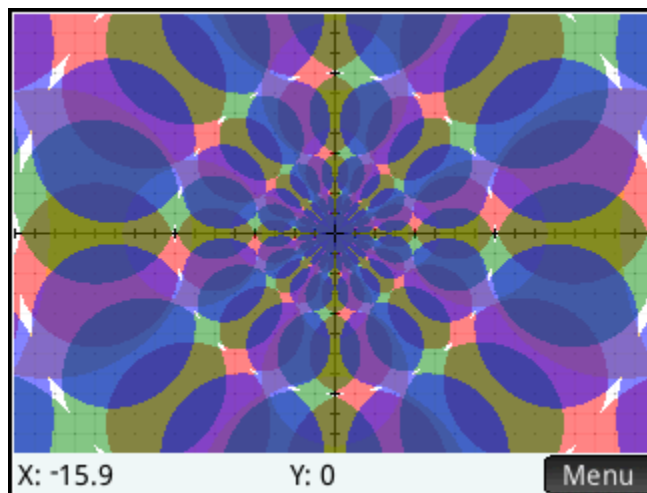
As you continue to press  to zoom in, you are zooming in on both the graph of $F1(X)$ at $x=0$ and the row of the $F2(X)$ -values where $x=0$. Connecting both of these simultaneously helps students see that:

- $F1(X)$ begins to exhibit local linearity near $x=0$
- The slope of $F1(X)$ near $x=0$ appears to be converging to 1 as a limit



Example: Color blending with multiple relations plotted in the Advanced Graphing app

The graphic below was created using 4 relations plotted in the Advanced Graphing app. It shows how color blending is used with multiple relations in the app. Change the color of each relation and observe the effect on the graph. The relations are shown below. If you are using the HP Prime virtual calculator, you can copy and paste them easily.



S1:

$$\begin{aligned} & (\sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (\pi/12)) \bmod (\pi/4)) - \\ & (\pi/8)) + \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (\pi/12)) \bmod (\pi/4)) - (\pi/8))^2 - \\ & 8}))) * \sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (\pi/12)) \bmod (\pi/4)) - (\pi/8)) - \\ & \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (\pi/12)) \bmod (\pi/4)) - (\pi/8))^2 - 8})))) < 0 \end{aligned}$$

S2:

$$\begin{aligned} & (\sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (-\pi/12)) \bmod (\pi/4)) - \\ & (\pi/8)) + \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (-\pi/12)) \bmod (\pi/4)) - (\pi/8))^2 - \\ & 8}))) * \sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (-\pi/12)) \bmod (\pi/4)) - (\pi/8)) - \\ & \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (-\pi/12)) \bmod (\pi/4)) - (\pi/8))^2 - 8})))) < 0 \end{aligned}$$

S3:

$$\begin{aligned} & (\sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (0/12)) \bmod (\pi/4)) - \\ & (\pi/8)) + \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (0/12)) \bmod (\pi/4)) - (\pi/8))^2 - \\ & 8}))) * \sin((\pi/\ln(2)) * \ln(\sqrt{X^2+Y^2}) * (3 * \cos(((\text{ATAN}(X/Y) + (0/12)) \bmod (\pi/4)) - (\pi/8)) - \\ & \sqrt{9 * \cos(((\text{ATAN}(X/Y) + (0/12)) \bmod (\pi/4)) - (\pi/8))^2 - 8})))) < 0 \end{aligned}$$

S4:

$$\begin{aligned} & (\sin((((\pi/\ln(2)))) * \ln(2 * \sqrt{X^2+Y^2}) * (6 * \cos(\text{ATAN}((((\min(\text{ABS}(X), \text{ABS}(Y)) / \max(\text{ABS}(X), \text{ABS}(Y)))))))) + \sqrt{(3 \\ & 6 * \cos(\text{ATAN}((((\min(\text{ABS}(X), \text{ABS}(Y)) / \max(\text{ABS}(X), \text{ABS}(Y))))))))^2 - \\ & 27}))) * \sin((((\pi/\ln(2)))) * \ln(2 * \sqrt{X^2+Y^2}) * (6 * \cos(\text{ATAN}((((\min(\text{ABS}(X), \text{ABS}(Y)) / \max(\text{ABS}(X), \text{ABS}(Y)))))))) \\ &) - \sqrt{(36 * \cos(\text{ATAN}((((\min(\text{ABS}(X), \text{ABS}(Y)) / \max(\text{ABS}(X), \text{ABS}(Y))))))))^2 - 27})))) < 0 \end{aligned}$$