

HP 39/40gs Math Activities Workbook



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For use with the HP 39/40gs Graphing Calculators



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Hewlett-Packard Company
16399 West Bernardo Drive
MS 66U-785
San Diego, CA 92127-1899
USA

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1 Birth Weights



Author: Sharon Taylor

Introduction

Data are all around us. Your age, height, and weight provide data about you. The number of miles, how fast you travel, and the type of transportation are all data that describe your trip to or from school. One way we analyze data is through the use of descriptive statistics. We use the term, “descriptive statistics,” because we can compute numbers that describe a data set. The most common statistics are mean, median, mode, and range.

- **Mean:** add all the values in your data set and divide by the number of data points you have.
- **Median:** once data are ordered from smallest to largest, find the middle data value. If the data set has an odd number of values, the median is the middle data point. If the data set has an even number of values, the median is found by adding the two data points in the middle and dividing by two.
- **Mode:** the most common value in the data set. If no value occurs more than once, there is no mode. If more than one value occurs most often, it is possible to have more than one mode.
- **Range:** describes how spread out the data is, and can be found by subtracting the lowest value from the highest value.

Activities

Exercise 1:

The birth weights from 11 of Donna's family members are given below. Each baby's weight has been converted from pounds and ounces to a decimal representation in pounds.

9.125 8.125 7.75 8.75 9.25 9.125 6.125 5.125 7.125 6.25 5.75

Find the mean, median and range.

To use the 39gs to find these values, start the Statistics Aplet. Press the **NUM** key to enter your data. If there are data that need to be cleared, press **2nd DEL** and choose the data you would like to delete. Enter your data into **C1**. Make sure you see the **1VAR** (instead of **2VAR**) tab at the bottom of your screen. When you press **STAT**, you will see the descriptive statistics for the data set, except for range. However, you will see the lowest value and the highest value. You can press **HOME** and perform the subtraction to find the range.

Exercise 2:

Now look at the weights of 11 children from Sharon's family. As you look at the data, make predictions about the mean, median, and range and compare them to Donna's family.

6.6875 5.75 7.875 9.875 8.375 7.75 4.5625 5.5 7.75 6.875 11.5

Compute the mean, median and range. How do they compare with your predictions?

To enter the data into the 39gs, make sure you are still in the Statistics Aplet and press **NUM**. You can enter the data for Sharon's family into **C2**. To make sure you can see the descriptive statistics for both data sets, press **SYMB** and check **H2**. Now when you go back to the **NUM** screen and press **STAT**, you will see the descriptive statistics for both sets of data.

Exercise 3:

Compare the mean, median and range for the two data sets. What observations are you able to see?

Extension 1: Is it possible to have two data sets with the same mean, but different medians? Show how this could be accomplished, or why it is not possible.

Extension 2: Is it possible to have two data sets with the same median, but different means? Show how this could be accomplished, or why it is not possible.

Extension 3: Is it possible to have two data sets with the same range, but different means? Show how this could be accomplished, or why it is not possible.

Teacher Notes

These activities focus on measures of centrality and spread. A series of exercises compares different data sets in which either mean or median is the same. This gives the class opportunities to note that very different data sets may have one measure of centrality the same, but may have one that is not, etc.

Teaching Standards

The following teaching standards can be found in this activity. These standards are general statements of what students should know and be able to do. The tables below include the general statement of each standard as well as its correlation to the Texas Essential Knowledge and Skills (TEKS) and the California Content Standards for Mathematics.

Calculate, use, and interpret measures of center and spread, including the mean, median, mode and range of a set of data.		
CA	Grade 6	1.1: Compute the range, mean, median, and mode of data sets.
	Statistics, Data Analysis, and Probability	1.4: Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.
	Grade 7, Statistics, Data Analysis, and Probability	1.3: Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.
	Grades 8-12, Probability and Statistics	6.0: Students know the definitions of the mean, median, and mode of a distribution of data and can compute each in particular situations.
TX	Grade 7	7.12(A): Describe a set of data using mean, median, mode, and range.
	Algebra 1	7.12(B): Choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.
	Grade 8	8.12(A): Select the appropriate measure of central tendency or range to describe a set of data and justify the choice for a particular situation.

Understand the correspondence between data sets and their graphical representations; select and use appropriate representations for presenting and displaying data, especially histograms, box plots, and scatterplots.		
CA	Grade 7, Statistics, Data Analysis, and Probability	1.1: Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.
	Grades 8-12, Probability and Statistics	8.0: Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.
TX	Grade 8	8.12(C): Select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem-and-leaf plots, circle graphs, bar graphs, box-and-whisker plots, histograms, and Venn diagrams, with and without the use of technology.

Using the HP 39/40gs

When the data is entered into the calculator, the screen should match the one shown in Figure 1-1. Make sure that **1VAR** is chosen. If your screen shows **2VAR**, simply press the key and the screen should change to **1VAR**.

n	C1	C2	C3	C4
7	6.125	4.5625		
8	5.125	5.5		
9	7.125	7.75		
10	6.25	6.875		
11	5.75	11.5		
12		
EDIT INS SORT BIG 1VAR STATE				

Figure 1-1

On the calculator, you can see the descriptive statistics for the two data sets. Notice that the sum of the data values and the number of data points are the same; as a result, the mean is the same.

1-VAR	H1	H2
NΣ	11	11
TOTΣ	82.5	82.5
MEANΣ	7.5	7.5
PVARΣ	2.059659	3.583097
SVARΣ	2.265625	3.941406
PSDEV	1.495151	1.892407
11		
OK		

Figure 1-2

1-VAR	H1	H2
SSDEV	1.505199	1.985298
MINΣ	5.125	4.5625
Q1	6.125	5.75
MEDIAN	7.75	7.75
Q3	9.125	8.375
MAXΣ	9.25	11.5
9.25		
OK		

Figure 1-3

The measures of variability (or dispersion) are different for the two data sets. Although we only looked at range here, the standard deviation and variance are also different. The term “variability” describes how “spread out” data values are. When all the values are pretty close to one another, the data have low variability (range, standard deviation, variance). When the data values are far apart from one another, there is high variability (range, standard deviation, variance). In addition to the range values being different for these data sets, you can look at the differences among **PVARΣ**, **SVARΣ**, **PSDEV**, and **SSDEV** on the calculator screen.

These are actual weights from two families. They are taken from larger data sets, and were very carefully selected so that the mean and median would be the same.

Answer Key

Exercise 1:

Donna's family:
mean = 7.5
median = 7.75
range = 4.125

Exercise 2:

Sharon's family
mean = 7.5
median = 7.75
range = 6.9375

Exercise 3:

The mean and median are the same, but the ranges are different. Although the data sets look very different (mainly due to the range of data), these two measures of central tendency are the same, which is usually an unexpected occurrence for many students.

Extension 1:

It is possible to have the same mean, but different medians. Just because one measure of central tendency is the same for two data sets, it does not imply that all measures of central tendency are all the same.

One way to show this is to use the existing data sets. If you subtract .5 from the median in one data set and add .5 (or any other value) to another value in that same data set, you keep the same sum and mean, but have a different median.

You can also create data sets in which the totals of the data values are the same, but the middle values are not the same.

Here's a simple example:

Data Set #1: 20 50 80	Mean = 50 and Median = 50
Data Set #2: 10 60 80	Mean = 50 and Median = 60

Extension 2:

It is possible to have the same median, but different means. We can again use the existing data sets to illustrate. Recall the reason the means are the same is because the sum of the data points and the number of data points are the same. Simply change one value (other than the median) in one data set to get different means. You can also create two new data sets in which the middle values are the same, but the sum of data points is different. Here's a simple example:

Data Set #1: 20 50 80	Mean = 50 and Median = 50
Data Set #2: 40 50 90	Mean = 60 and Median = 50

Extension 3:

It is possible to have the same range, but different means. To ensure the same range, the data sets must have the same lowest and highest values. As with Extension 2, to have different means, simply make sure the sum of the data points is different.

Here's a simple example:

Data Set #1: 20 50 80	Mean = 50 and Range = 60
Data Set #2: 30 60 90	Mean = 60 and Range = 60

2 Matching Data and Graphs



Author: Sharon Taylor

Introduction

Often data sets are presented and we are asked to create a graphical representation of that data. Seldom are we given a graph and asked about an original data set. This activity will explore several data sets and graphs, specifically box-and-whisker plots and histograms.

Box-and-whisker plots are created from a five-number summary. Those five numbers are:

- lowest (or minimum)
- lower (or first) quartile
- median
- upper (or third) quartile
- highest (or maximum)

The lowest and highest values are easy to find once the data set has been ordered. The median is the value exactly in the middle of an ordered set of data. There are just as many data points above the median as there are below. The lower (or first) quartile is the median of the lower half of the data. The upper quartile is the median of the upper half of the data. The term *quartile* comes from the fact that when you take half of the lower half of data and then take half of the upper half, you have created four parts, or quarters, in the data.

The five-number summary is represented graphically with short vertical lines using a "box" and "whiskers." The box in the middle of the graph represents three values. The left side of the box is the lower quartile, the vertical line inside the box is the median, and the right side of the box is the upper quartile. From each side of this box are horizontal lines that are called the "whiskers." The short vertical line at the end of the left whisker is the lowest value, and the short vertical line at the end of the right whisker is the highest value. A sample box-and-whisker plot is shown below.

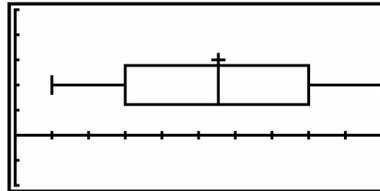


Figure 2-1

Histograms group data into equal intervals. Once you have decided how you want to group your data (by 10's or 100's and so on), you tally how many data points fall into each interval. The x-axis is marked off using the intervals you have chosen. The y-axis represents the frequency. A vertical bar is extended upward from an interval on the x-axis to show the frequency for that interval. A sample histogram is shown below.

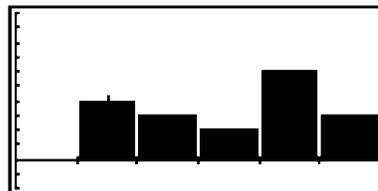


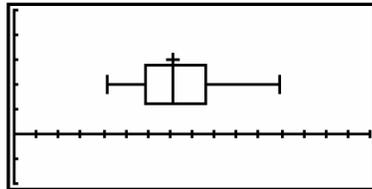
Figure 2-2

Exercise 1

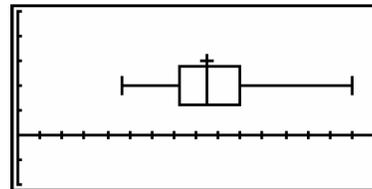
1. The following data represent enrollments at 20 middle schools in San Antonio.

688	512	1197	629	927
799	588	418	650	595
875	532	869	1192	854
685	822	746	572	863

Two box-and-whisker plots are shown below. The x - axis goes from 0 to 1600 with a scale of 100. Which of these plots represents the data above? How do you know?



Plot # 1



Plot # 2

2. Enter the data into your 39gs, create the box-and-whisker plot, and verify your answer.

You enter data as you did in the previous activity. To produce a graph, you will need to go to the **PLOT SETUP** menu (2nd PLOT). Choose the box-and-whisker plot as your graph and then enter the settings for the minimum and maximum x and y values and scale. (For a box-and-whisker plot, you will not need to worry about **HWIDTH** or **HRNG**.) You will need to go to the **SYMB** menu and check **H1** in order to see the graph. Press **PLOT** and the box-and-whisker plot will appear on the screen.

3. Estimate how many students are in your school. Where is your estimate on the box-and-whisker plot? Explain your answer.

Extension 1

Estimate the five-number summary for the second box-and-whisker plot in Exercise 1.

Exercise 2

1. A box-and-whisker plot displaying the number of teachers at middle schools in a San Antonio districts is below. The x-axis for the graph is marked off in increments of 10.

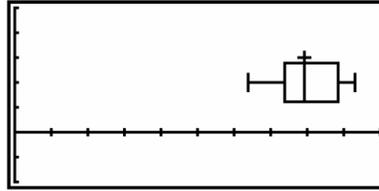


Figure 2-3

Data sets for three school districts are given below. Which district matches the above box-and-whisker plot? How do you know?

District #1	District #2	District #3
67	93	42
98	86	32
100	66	65
66	80	39
63	79	51
66	87	51
79	74	38
58	74	37
69	64	51
73	90	35
	77	50
	93	68
		54
		39
		49

2. Confirm your choice by creating the box-and-whisker plot on your 39gs.
3. What are the characteristics of the other two data sets?
4. You may have noticed that the number of data points in each set was different. Explain why the number of data points is not an issue when comparing box-and-whisker plots.

Exercise 3

A histogram for the number of calories for 10 popular items at a fast food restaurant is given below. In the graph, the x-axis is marked off in increments of 100 calories and the y-axis is marked off in increments of 1 item.

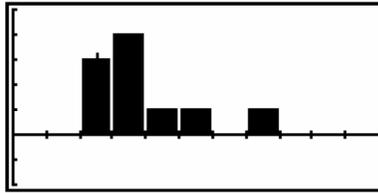


Figure 2-4

- Describe what you see in this histogram.
- Below is data for number of calories for the 10 popular items from three different fast food restaurants.

	Restaurant #1	Restaurant #2	Restaurant #3
Signature Burger	670	620	540
Regular Hamburger	290	310	250
Regular Cheeseburger	330	355	300
Chicken Sandwich	660	510	360
Chicken Nuggets	210	580	250
Medium Fries	360	400	380
Medium Coke	200	300	210
Chocolate Shake	690	1070	770
Fish Sandwich	630	490	380
Sausage Biscuit	390	540	400

Which restaurant is shown in the histogram? How do you know?

You can use the 39gs to create a histogram by following steps similar to those of setting up a box-and-whisker plot. Once the data has been entered, go to **PLOT SETUP**, choose histogram, and input the appropriate x and y settings. To graph a histogram, you will need to set values for **HWIDTH** and **HRNG**. **HWIDTH** is set to be how wide you want the bar to be. **HRNG** is typically set the same as your **XRNG**.

- When comparing box-and-whisker plots, the number of observations in a data set did not have to be the same. Explain why it is important when comparing histograms.

Extension 2

Create histograms for the other two data sets in Exercise 3.

Exercise 4

The gas mileage (given in miles per gallon) for 18 vehicles appears below.

20	15	40	41	28	36	34	16	31
35	33	38	27	34	19	29	17	18

There are two graphs shown below. In both graphs, the x-axis is marked in increments of 10 and the y-axis is marked in increments of 1. Which one matches the data set? How do you know? Use your 39gs to confirm your choice.

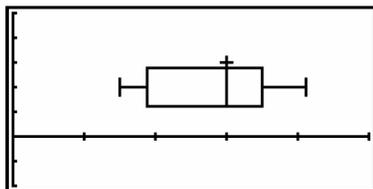


Figure 2-5

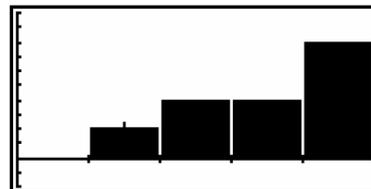


Figure 2-6

Teacher Notes

These activities require students to make inferences and convincing arguments based on an analysis of given or collected data, and select and use appropriate representations for presenting and displaying data that include histograms and box-and-whisker plots.

TEKS

The following TEKS can be found in this activity. The TEKS are listed in grade-level order, not the order in which they appear in the activity. Also, some of the TEKS statements are abbreviated or paraphrased here.

- 7.11B: make inferences and convincing arguments based on an analysis of given or collected data
- 8.12C: select and use an appropriate representation for presenting and displaying data

Exercise 1

The first box-and-whisker plot is correct. The data shown in the exercise are from 17 SAISD schools and 3 South San ISD schools.

The data that create the second box-and-whisker plot also represent enrollments at 20 middle schools in San Antonio, but from other districts (Alamo Heights, East Central, Harlandale, and Judson).

The data are:

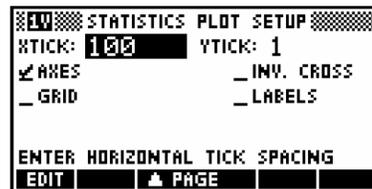
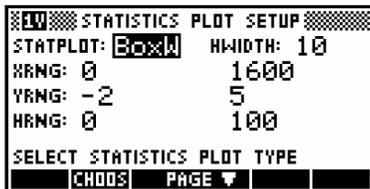
1015	1209	663	463	481	536	833	913	723	831
998	1503	952	854	998	843	975	916	810	948

For reference, the values used in each plot are below.

The five-number summary for Plot #1 is: 418 591.5 717 866 1197

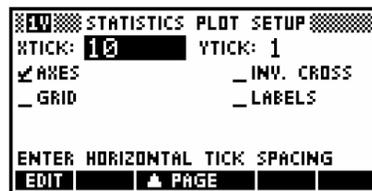
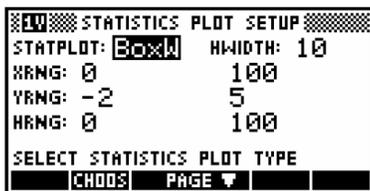
The five-number summary for Plot #2 is: 463 766.5 883.5 986.5 1503

The **PLOT SETUP** screens used to create the box-and-whisker plots are shown below.

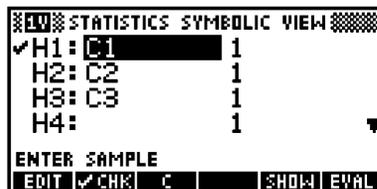


Exercise 2

To obtain the box-and-whisker plots for all three data sets, you can use the same **PLOT SETUP** screens.

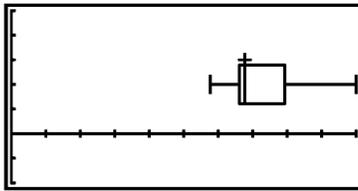


To obtain each individual box-and-whisker plot, go into **SYMB** and indicate which data set you would like to plot.

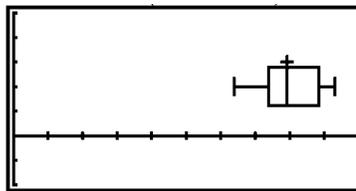


The box-and-whisker plot in the exercise is for District #2, which is Northside ISD. District #1 is Northeast and District #3 is San Antonio ISD. All three plots are shown below. The x - axis is marked off in increments of 100.

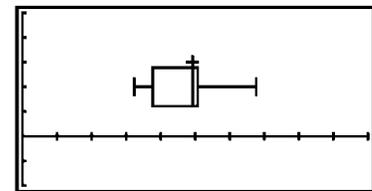
District #1 (Northeast)



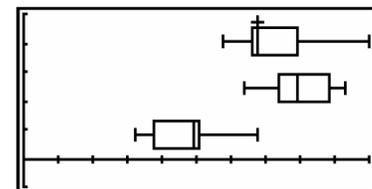
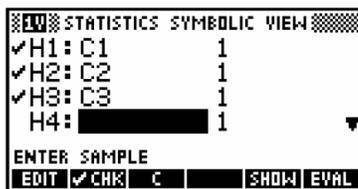
District #2 (Northside)



District #3 (San Antonio)

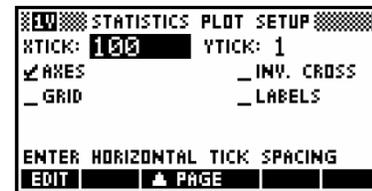
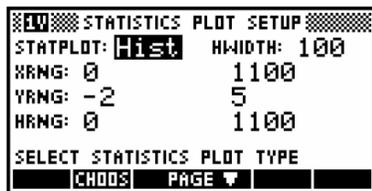


By checking all three data sets in the **SYMB** menu, you can see all three box-and-whisker plots on one screen. To prevent the last plot from touching the x - axis, you should make a slight modification to the **PLOT SETUP** menu as shown below.

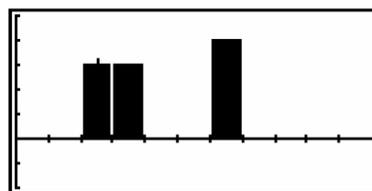
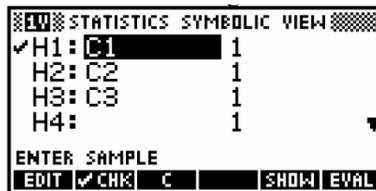


Exercise 3

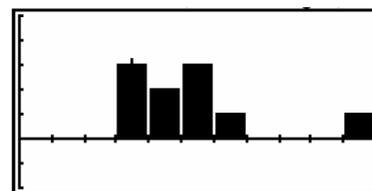
Restaurant #1 is Burger King. Restaurant #2 is Whataburger. Restaurant #3 is McDonalds, which matches the histogram. To see the histograms for all three restaurants, use the same **PLOT SETUP** screens shown below.



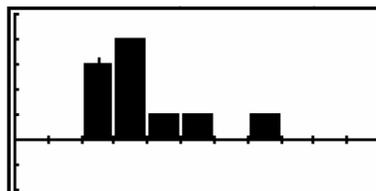
To see each histogram, check the appropriate data set in the **SYMB** menu.



Restaurant #1 (Burger King)



Restaurant #2 (Whataburger)



Restaurant #3 (McDonalds)

As shown in the **PLOT SETUP** menu, the x - scale for each histogram is 10, and the y - scale is 1.

Exercise 4

The make and model of the vehicles is given in the table below. The data are for 2007 model vehicles. The correct graph in the exercise is the box-and-whisker plot.

Make and Model of Car	Highway Mileage
Toyota Corolla	41
Toyota Yaris	40
Honda Civic	38
Hyundai Elantra	36
Hyundai Accent	35
Nissan Altima	34
Nissan Versa	34
Toyota Camry	33
Toyota Avalon	31
Chevrolet Impala	29
Pontiac G6	28
Ford Taurus	27
Dodge Charger	20
Chevrolet Suburban	19
Lincoln Navigator	18
Toyota Land Cruiser	17
Mercedes Benz G500	16
Jeep Grand Cherokee	15

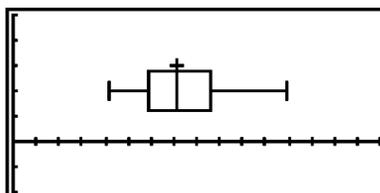
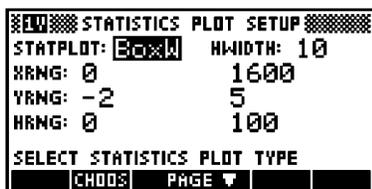
The data used for the histogram comes from the grades of 18 students on a 50-point project in a statistics class. That data set is below.

18 18 20 22 23 25 30 34 35
38 40 42 42 43 45 46 48 49

Answer Key

Exercise 1

- Plot #1 is correct. Students should be able to justify their answers by looking at the minimum and maximum values in the given data set and matching them to the correct box-and-whisker plot.
- To create the box-and-whisker plot on the calculator, use the **SETUP** screens below. The correct plot is also shown again below.



- If students do not have a feel for how many students are in your school, they can still make a guess. The emphasis should be placed on locating values on the box-and-whisker plot rather than their estimation of the number of students.

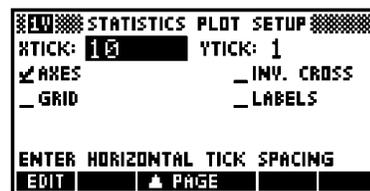
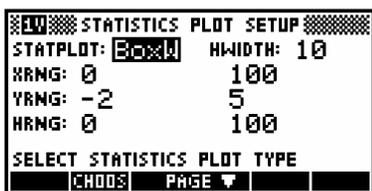
Exercise 2

- The box-and-whisker plot is for District #2. Students should be able to confirm their choice by using the calculator to create the plots, by using five-number summaries for the data, or simply relying on the highest and lowest values from the data sets.

The five-number summaries for each district are given below.

District #1:	58	66	68	79	100
District #2:	64	74	79.5	88.5	93
District #3:	32	38	49	51	68

- The **SETUP** screens for creating the box-and-whisker plot are shown below. The correct graphs for all districts were previously shown in the *Teacher Notes* section above.



- For District #1, the lowest value is between 50 and 60, and the highest value is at 100. For District #3, the lowest value is between 30 and 40, and the highest value is between 50 and 60. You may also want students to describe the data sets using the five-number summaries, which were given in the answer to #1 above.
- The number of data points is not an issue when comparing box-and-whisker plots. We are only comparing low, middle and high values, not the actual number of data points. We will do this with histograms in the next exercise.

Exercise 3

- Students should see in the histogram that there are 3 observations in the 200 bar, 4 observations in the 300 bar, and one observation each in the 400 bar, 500 bar, and 600 bar.

- The restaurant that matches the histogram is Restaurant #3, which is McDonalds. Students can verify their answer by counting tallies of all three data sets to see what matches and by creating histograms on the calculator.
- To choose between two histograms for one data set, the two histograms should have the same number of data points represented. Otherwise, if one data set has more points, its histogram will have taller bars (to represent a higher frequency), and you can easily tell which histogram is correct by simply counting data points without really understanding the graph.

Exercise 4

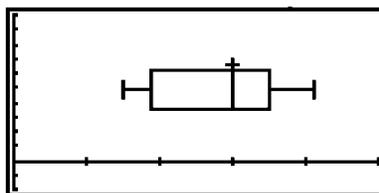
Students should notice that the data set has 5 observations that would fall in the 10 bar of a histogram, 4 observations in the 20 bar, 7 observations in the 30 bar, and 2 observations in the 40 bar. These numbers do not match the histogram shown. On the other hand, the five-number summary of the data is:

15 19 30 35 41

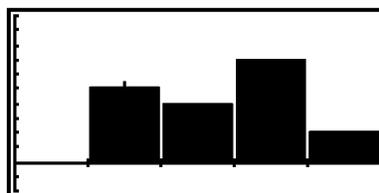
which corresponds to the box-and-whisker plot shown.

Students should also create both graphs for the data set on the calculator.

The box-and-whisker plot for the given data set is the one shown in the exercise.



The histogram for the given data set is shown below.



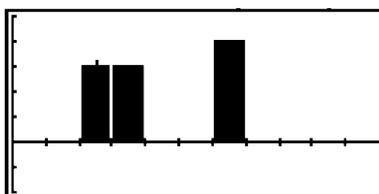
Extension 1

The five number summary for Plot #2 is: 463 766.5 883.5 986.5 1503

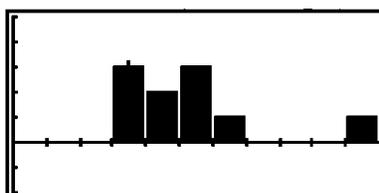
These are the exact values and students will not be able to find them easily. However, they should be able to make reasonable estimates by reading the box-and-whisker plot.

Extension 2

The two histograms for Restaurant #1 and Restaurant #2 are shown again below.



Restaurant #1 (Burger King)



Restaurant #2 (Whataburger)

Many times when working with sequences, we describe the pattern we see in terms of the previous term. The way we represent the previous term in the sequence named u is to use the notation $u(n-1)$. With the sequence we are looking at here, we are adding two to the previous term. We can represent that at: $u(n) = u(n - 1) + 2$.

In addition to describing patterns as sequences and in words, we also use tables.

4. Create a table to represent your sequence.

Figure Number	Number of Squares
1	
2	
3	
4	
5	

5. Do you see the same pattern in the table that you saw when you listed the sequence? Use this pattern to predict the number of squares for Figures 6 and 7.

Let's create this table on the 39gs. Start the **Sequence** Aplet. You will be prompted to input the first and second terms. You will then need to enter an expression for the sequence. You can use the $u(n)$ we defined earlier in this exercise. Once you input this information in the **SYMB** menu, you can press **NUM** and see the table of values.

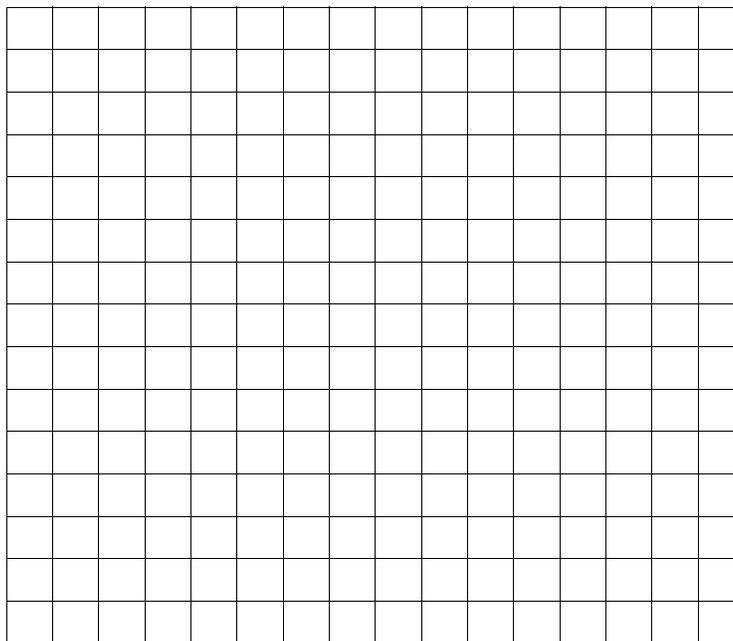
Verify your values for the 6th and 7th figures in your table are the same as what you see on the 39gs.

The sequence you have created here is an arithmetic sequence. A sequence is arithmetic if it has a common difference. You can find a common difference by subtracting two consecutive terms. It is called a *common* difference, because regardless of which two consecutive terms you subtract, you will always get the same answer. The common difference in our sequence is two.

Exercise 2

In Exercise 1, we looked at a picture pattern and then used that to create a sequence of numbers. We also looked at our sequence in a table format, both on paper and on the calculator. Another way to look at sequences is with a graph.

1. Use your table from Exercise 1 to create a graph of your data.



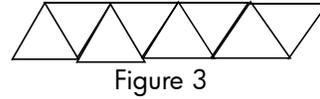
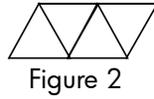
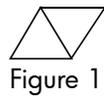
Make sure the values on your graph match the values in the table as well as your predicted values for Figures 6 and 7.

You already have all of your sequence information entered in your 39gs. By going to **PLOT SETUP** and inputting the appropriate settings, you can press **PLOT** and see your graph on the calculator.

2. You now have a graph of an arithmetic sequence. If you were to connect the points, what kind of graph would you have?
3. How could you use this connected graph to continue to make predictions?
4. How does the calculator help you make predictions?

Exercise 3

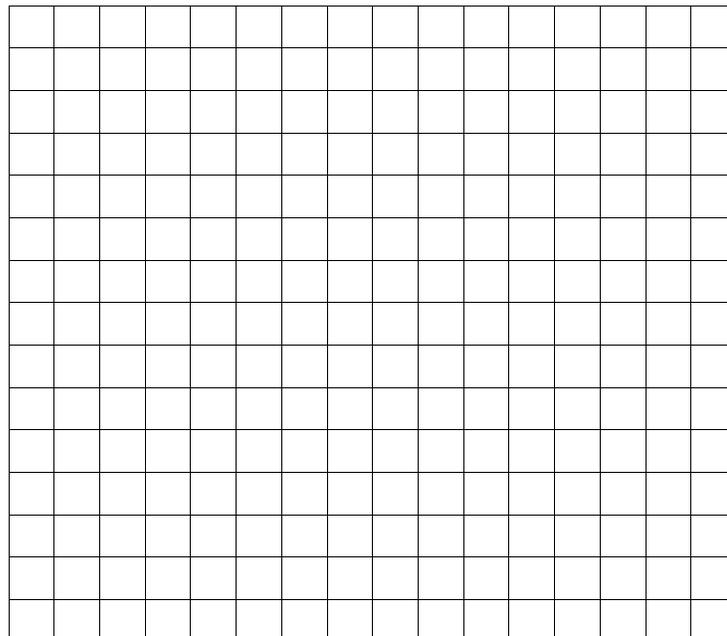
Below is another picture pattern. Look for a pattern and draw or create Figure 4.



- Count the number of triangles in each figure and list them in a sequence.
- Create a table using the figure number and the number of triangles in the figure.

Figure Number	Number of Triangles
1	
2	
3	

- What pattern do you see in the sequence and table? Describe what you see.
- Use your information from the table to create a graph of your sequence



5. Is this graph linear like the sequence from Exercises 1 and 2?

What you have created is a geometric sequence. Rather than having a common difference, as with the arithmetic sequence, geometric sequences have a common ratio. Based on how you computed a common difference, how do you think you will compute a common ratio? What is the common ratio for the sequence in this exercise?

Extension 1

Create a sequence, table, and graph for the number of segments needed to create each figure. Do the same for the perimeter of each figure.

Extension 2

The area (number of triangles) is geometric. Create a sequence, table, and graph for the number of segments. Describe what you see in words. What type of sequence do you think you have?

Extension 3

In the activity "Perimeter and Area of Similar Shapes," you looked at similar squares, rectangles, and circles, created tables with dimensions and perimeter and circumference, and the differences in perimeter and circumference. How are the differences for similar figures related to the sequences created in this activity?

Go back and look at similar figures and tables. Look at the differences in the columns from the previous activity and make connections.

Teacher Notes

This activity requires students to use tables and graphs to represent and describe proportional relationships and letters to represent unknown quantities in equations. They also generate formulae and describe the relationship between terms in an arithmetic sequence using words and symbols.

Many teachers have tried toothpicks as a hands-on approach to building the figures and have been very successful. However, others have found that Q-tips work better, since they do not have sharp ends.

It is important to have students verbalize the patterns they see in the sequences. This is a direct link to the symbolic representation used in a recursive definition of a sequence.

The notation for sequences is very difficult for many students. Emphasize that u is simply the name of the sequence. The letter n is just a way to represent any term in the sequence. When we talk about the first term, $n = 1$. When we talk about the 12th term, $n = 12$, the biggest difficulty is often in talking about "the previous term." The $u(n - 1)$ notation is cumbersome to quite a few students. However, since describing patterns in terms of the previous terms is the easiest way for students to begin their approach to sequences, the notation, though difficult, is necessary.

The common difference concept in arithmetic sequences will develop the idea of slope in linear functions. Students can notice the table shows common differences in x and y , and, in the graph, the height of the steps are the same size.

The exercises have been set up so that students experience as many representations as possible: concrete, numeric, table, graph, verbal and written.

The following TEKS can be found in this activity. The TEKS are listed in grade-level order, not the order in which they appear in the activity. Also, some of the TEKS statements are abbreviated or paraphrased here.

- 6.4A: use tables and symbols to represent and describe proportional and other relationships such as those involving arithmetic sequences
- 6.5: use letters to represent an unknown in an equation
- 7.4A: generate formulas
- 7.4B: graph data to demonstrate relationships in familiar concepts
- 7.4C: use words and symbols to describe the relationship between the terms in an arithmetic sequence and their positions in the sequence
- 8.3A: compare and contrast proportional and non-proportional linear relationships
- 8.4: make connections among various representations of a numerical relationship

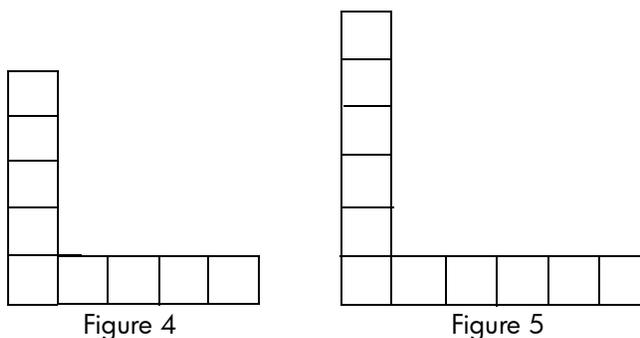
Table 1: TEKS Covered, by Activity

Activity	TEKS Covered
Exercise 1	6.4A, 6.5, 7.4A, 7.4C, 8.4
Exercise 2	7.4B, 8.4
Exercise 3	6.4A, 6.5, 7.4A, 7.4B, 7.4C, 8.3A, 8.4
Extension 1	6.4A, 6.5, 7.4A, 7.4B, 7.4C, 8.4
Extension 2	6.4A, 6.5, 7.4A, 7.4B, 7.4C, 8.3A, 8.4
Extension 3	6.4A, 6.5, 7.4A, 7.4B, 7.4C, 8.3A, 8.4

Answer Key

Exercise 1

1.



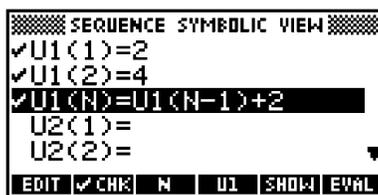
Students can use manipulatives to create the 4th and 5th figures. Their descriptions will probably vary. Some may talk about the number of squares needed, others may describe their pattern based on how many "sticks" or manipulatives they needed, while others may talk about adding squares to the top and the side. As long as the description is correct, students have a lot of latitude with their description in this part. However, once the question has been asked about the number of squares in the figure, there is a precise answer students should provide.

2. The list of numbers is 3, 5, 7, 9, 11.
3. The pattern is that you are adding two each time. Try to get students working with the terminology that they are adding two to the previous term.
- 4.

Figure Number	Number of Squares
1	3
2	5
3	7
4	9
5	11
6	13
7	15

5. The same pattern holds by adding two to the previous term.

To see the table in the 39gs, use the following input screen:



By pressing **NUM**, you should see the table of values for the sequence.

N	U1		
2			
4			
6			
10			
12			

200M BIG DEFN

To see additional values in the table, you can scroll down.

N	U1		
10			
12			
14			
16			
18			
20			

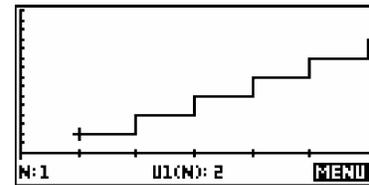
200M BIG DEFN

Exercise 2

1.

SEQUENCE PLOT SETUP	
SEQPLOT:	Stairstep
NRNG:	1 6
XRNG:	0 6
YRNG:	-3 15
CHOOSE SEQUENCE PLOT TYPE	
CHOOSE	PAGE

SEQUENCE PLOT SETUP	
XTICK:	1 YTICK: 1
<input checked="" type="checkbox"/> SIMULT	<input type="checkbox"/> INV. CROSS
<input checked="" type="checkbox"/> AXES	<input type="checkbox"/> LABELS
<input type="checkbox"/> GRID	
ENTER HORIZONTAL TICK SPACING	
EDIT	PAGE



2. If the points are connected, you would use a line.
3. If you extend the line, you can use it to make predictions.
4. The calculator helps make predictions by showing values on the graph and the table.

Exercise 3

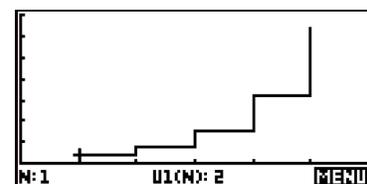
1. The sequence for the number of triangles is 2, 4, 8.
- 2.

Figure Number	Number of Triangles
1	2
2	4
3	8
4	16

3. You are multiplying by 2 each time. Another way to phrase it is to say that you are multiplying the previous term by 2.
4. **SETUP** screens to see the graph on the calculator are shown below.

SEQUENCE PLOT SETUP	
SEQPLOT:	Stairstep
NRNG:	1 5
XRNG:	0 6
YRNG:	-5 35
CHOOSE SEQUENCE PLOT TYPE	
CHOOSE	PAGE

SEQUENCE PLOT SETUP	
XTICK:	1 YTICK: 5
<input checked="" type="checkbox"/> SIMULT	<input type="checkbox"/> INV. CROSS
<input checked="" type="checkbox"/> AXES	<input type="checkbox"/> LABELS
<input type="checkbox"/> GRID	
ENTER HORIZONTAL TICK SPACING	
EDIT	PAGE



5. This graph is not linear. The sequence in this exercise is geometric. Geometric sequences do not produce linear graphs, but rather exponential graphs.

Extension 1

Number of segments: 10, 16, 22

```

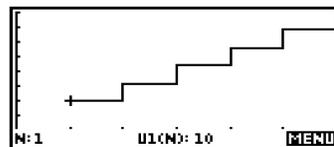
SEQUENCE SYMBOLIC VIEW
✓U1(1)=10
✓U1(2)=16
✓U1(N)=U1(N-1)+6
U2(1)=
U2(2)=
EDIT ✓CHK SHOW EVAL
    
```

N	U1		
1	10		
2	16		
3	22		
4	28		

10
ZOOM BIG DEFN

```

SEQUENCE PLOT SETUP
SERPLOT: Stairstep
NRNG: 1 6
XRNG: 0 6
YRNG: -5 40
CHOOSE SEQUENCE PLOT TYPE
CHOOSE PAGE
    
```



Perimeter: 8, 12, 16

```

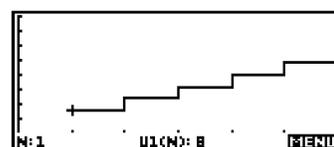
SEQUENCE SYMBOLIC VIEW
✓U1(1)=8
✓U1(2)=12
✓U1(N)=U1(N-1)+4
U2(1)=
U2(2)=
EDIT ✓CHK SHOW EVAL
    
```

N	U1		
1	8		
2	12		
3	16		
4	20		

ZOOM BIG DEFN

```

SEQUENCE PLOT SETUP
SERPLOT: Stairstep
NRNG: 1 6
XRNG: 0 6
YRNG: -5 40
CHOOSE SEQUENCE PLOT TYPE
CHOOSE PAGE
    
```



Extension 2

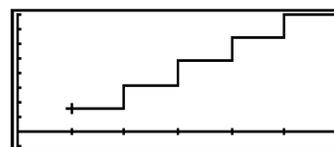
The sequence for the number of segments is 5, 9, 17. Students may need to draw an additional figure to come up with the next term. The sequence is neither arithmetic nor geometric, however, there is a pattern. Each time you are adding increasing powers of 2 (first you add 4, then 8, then 16). Although you are adding each time, you are not adding the same amount each time. Therefore the sequence is not arithmetic. Also, since you are not multiplying by the same number each time, the sequence is not geometric. The goal is to get students to realize that there are many sequences that will not fall into either category. Even an advanced student will have difficulty coming up with $u(n) = 3n^2 - 5.8n + 8$.

Extension 3

In the table for perimeter for similar squares, the common difference is 8. Students can write a recursive formula $u(n) = u(n - 1) + 8$. The symbolic input screen and graph are shown below.

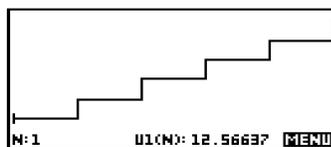
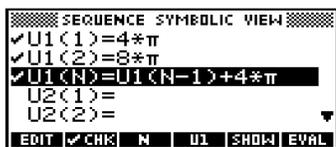
```

SEQUENCE SYMBOLIC VIEW
✓U1(1)=8
✓U1(2)=16
✓U1(N)=U1(N-1)+8
U2(1)=
U2(2)=
EDIT ✓CHK N U1 SHOW EVAL
    
```



In the table for perimeter for similar rectangles, the common difference is also 8. These similar rectangles have the same recursive formula and graph as similar squares. Make sure to point out that this relationship and common difference of 8 is only for the rectangles with the dimensions in the problem. If the dimensions are different, there will be a different common difference in the perimeters.

In the table for circumference for similar circles, the common difference is 4π . Students can write a recursive formula $u(n) = u(n - 1) + 4\pi$. The symbolic input screen and graph are shown below.



4 Predicting the US Population



Author: Michael Grasse

Introduction

What was the population of the United States in the year you were born? What will be the population of the US in the year you turn 18 years old? The US Census is held every ten years (1970, 1980, and so on), so if you weren't born in a year that ends in a zero, how can you answer the first question? Without a time machine, how can you answer the second question? The goal of this activity is to use existing US Census data and some mathematical techniques to build a model that will estimate the US population in the past, and then predict the population in the future.

Activity

The table below contains population data from the US Census over the last 30 years. An applet called *CensusData* containing this data and other settings will be beamed to your calculator. While you are waiting for it, look at the data and do exercises 1 and 2.

Table 4-1

Year	Population	Change from Previous Census
1970	203,210,158	23,886,983
1980	226,545,805	23,335,647
1990	248,709,873	22,164,068
2000	281,421,906	32,712,033

Exercises

1. After looking at the data, guess what the US population was in the year you were born.
2. Guess what the population will be in the year you turn 18.

Now plot the US Census data in the CensusData Applet. Press **APLET**, highlight **CensusData**, and press the **START** menu key (or **ENTER**). You should see the data from the first two columns in Table 4-1. The years will be in **C1** and the population in **C2**. If you do not have the CensusData Applet, just enter the data from Table 4-1 above.

To see these points plotted, just press **PLOT**. The window should already be set up to create a graph that contains the data and is easy to read. Your teacher will be projecting this same graph on the board. If you do not have the CensusData Applet, just enter the data from Table 4-1 above.



3. Do all the points look like they fit on a line?

Even if they don't all fit on a line, a line can be drawn on the board that best represents all the data. Either your teacher or one of you should draw a line on the board that comes the closest to all of the data points shown in the projection. Extend your line as far as you can. Your line is actually a graphical model for the population of the US. You can use this model to make some predictions: at the board, find the point on the horizontal axis that represents the year you were born. From that point, look straight up to the line, then look straight over to the vertical axis.

4. What does this graphical model predict the US population to be in the year you were born? Is this much different from your guess in exercise 1?
5. Use the same steps to predict the population of the US in the year that you turn 18. What is the prediction? Is this much different from your guess in exercise 2?

This process can be reversed also: at the board, find the point on the vertical axis that represents a population of 300,000,000. From that point look straight over to the line, then look straight down to the horizontal axis.

6. In what year does this graphical model predict the US population will reach 300,000,000?

The HP 39gs can also create a linear model for the population that will probably make more accurate predictions than the line drawn on the board. There is a mathematical process that can actually find the best line that fits all of the data. The HP39gs can boil this process down to a formula that you can use to make predictions. While you are looking at the data, press the **Menu** key and then the **Fit** key. How does the line that the HP 39gs draws compare with the one drawn by you or your teacher on the board?

Press **SYMB**. Use the down arrow key to highlight the **Fit1** blank and press **SHOW**. You might have to use the right arrow key to see all of it. This formula expresses the population (Y) in terms of the year (X). We can use this formula as a more accurate model.

7. Write down this formula by filling in the blanks (write all the numbers!). We will use **P** instead of **Y** so you remember it means population:

$$P = \underline{\hspace{2cm}} \bullet X - \underline{\hspace{2cm}}$$

Now enter this formula - an algebraic model for the US population- into the solve applet. Press the **APLET** key and choose **SOLVE**. You will see a list that can hold up to ten equations. Press **SHIFT DEL** (which is **CLEAR**) if necessary to clear out all of the equations. Now type the equation that you wrote above into **E1** (input all the numbers!). Note that one of the menu keys will input an equals sign (=). Now press the **NUM** key. You will see two input blanks, one for **P** (the population) and one for **X** (the year).

Use the arrow keys to highlight the **X** blank. Type the year in which you were born. Now use the arrow keys to highlight the **P** blank and press the **SOLVE** menu key.

8. What does this algebraic model predict the US population was in the year you were born? Is this much different from your prediction in exercise 4?
9. Use the same steps to predict the population of the US in the year that you turn 18. What is the prediction? Is this much different from your prediction in exercise 5?
10. Predict the population of the US when you turn 35 years old.

Use the arrow keys to highlight the **P** blank. Type in a population of **300,000,000**. Now use the arrow keys to highlight the **X** blank and press the **SOLVE** menu key.

11. In what year does this model predict the population of the US to be 300,000,000?
12. In what year does this model predict the population of the US to be 400,000,000?

Teacher Notes

This activity can be used as an introduction to mathematical modeling with linear functions. The emphasis here is on a conceptual understanding of using a line as a model and then refining that model by using the equation of the line to answer questions about the model situation.

Teaching Standards

The following teaching standards can be found in this activity. These standards are general statements of what students should know and be able to do. The tables below include the general statement of each standard as well as its correlation to the Texas Essential Knowledge and Skills (TEKS) and the California Content Standards for Mathematics.

Use tables, graphs, and equations to represent situations involving linear relationships.		
CA	Grade 7, Mathematical Reasoning	2.5: Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
TX	Algebra 1	A.1(B): Gather and record data and use data sets to determine functional relationships between quantities.
		A.2(C): Interpret situations in terms of given graphs or create situations that fit given graphs.
		A.5(C): Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Investigate situations and solve problems using linear equations.		
CA	Grade 7, Algebra and Function	4.0: Students solve simple linear equations and inequalities over the rational numbers.
CA	Grade 7, Mathematical Reasoning	2.3: Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
TX	Algebra 1	A.2(D): Collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.
		A.7(A): Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems.

Using the HP 39/40gs

This activity serves as an introduction to 2-variable statistics on the HP39gs. There are no formal extension exercises, but students may wish to use the algebraic model to interpolate or extrapolate answers to their own questions. Transferring and using a custom aplet is also featured here. Post-activity discussions might include an evaluation of graphical and algebraic models, sources of error, and the logistics of the US Census.

You may want to check the algebraic model against some population milestones such as:

- The US population in 1950 was 76,212,168
- The population reached 100 million in 1915
- The population first reached 200 million in 1967

When projecting the data on the board, you will have to draw and label the horizontal and vertical axes yourself. The window is set so that the horizontal axis goes from 1960 to 2050, and the vertical axis goes from 170 million to 420 million. When drawing the line, you may wish to draw it through the first three data points. Don't worry too much about being perfectly accurate, purely graphical representations are always somewhat limited.

n	C1	C2	C3	C4
1	1970	2.032E8		
2	1980	2.265E8		
3	1990	2.487E8		
4	2000	2.814E8		

1970

EDIT INS SORT BIG 2VAR=STATS

Figure 4-1

```

CENSUSDATA PLOT SETUP
XANG: 1960 2050
YANG: 1700000... 4200000...
S1MARK: + S2MARK: x S3MARK: +
S4MARK: : S5MARK: x
CHOOSE MARK FOR SCATTER PLOT
CHOOS PAGE

```

Figure 4-2

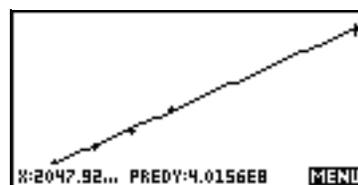


Figure 4-3

Figure 4-1 illustrates the data in the Numeric View of the CensusData Aplet.

Figure 4-2 illustrates the Plot Setup view of the aplet.

Figure 4-3 illustrates the scatter plot as well as the line of best fit. The tracer is on the first point where Y is greater than 400 million. The year indicated is late in 2047.

Web Resources

This site contains a wealth of information about the 2000 US Census and an enormous amount of data. Population figures from 1790 to 2000 can be found here:

<http://www.census.gov/main/www/cen2000.html>

Answer Key

1. These guesses may vary widely, but hopefully they are all between the appropriate given population values.
2. These guesses may also be quite varied, but they should all be above 281,421,906.
3. The first three points look very linear. The population differences are all about 23,000,000 for each of the first three decades. Note that the Census data for 1960 is not shown, but the change from the 1960 census to the 1970 census is accurate. The last point is above the line formed by the first three points. The population grew almost 33,000,000 from 1990 to 2000.
4. Answers may vary. These exercises should be done as a class.
5. Answers vary. These exercises should be done as a class.
6. Answers vary. These exercises should be done as a class.
7. $P = 2567993.12 X - 4357494407.7$
8. These answers all depend on the age of the student. Answers should not vary among students born in the same year.
9. These answers all depend on the age of the student. Answers should not vary among students born in the same year.
10. These answers all depend on the age of the student. Answers should not vary among students born in the same year.
11. The model predicts the population of the US to be 300,000,000 in the year 2008. It actually happened in 2006.
12. The model predicts the population of the US to be 400,000,000 in the year 2047. For comparison, the US Census projections place the US population at 391,946,000 in 2040.

5 Hispanic Population in the US



Author: Sharon Taylor

Introduction

The previous activity looked at population data for the United States. For this activity, we will focus on the Hispanic and overall population for the US. In the last activity, we used a linear model for the total US population. The table below shows the data from the last activity; the second column contains the total Hispanic population of the US for each census.

This activity will tie together two previous ideas you have worked with: sequences and statistics. First, you will explore the data using the ideas of common difference and common ratio from your work with sequences. You will then use the Statistics Aplet to see how accurate your sequence representations are by making predictions based on models created by your graphing calculator. You may also work with an aplet that has to be put on your calculator.

Table 5-1

Year	US Hispanic Population	US Total Population
1970	9,589,216	203,210,158
1980	14,608,673	226,545,805
1990	22,354,059	248,709,873
2000	35,305,818	281,421,906

Activities

Exercise 1: Model the US Population

In the Sequence Aplet, you can define up to ten different sequences. Press the **APLET** key and select the **Sequence** Aplet. Then press the **START** menu key to see the Sequence Symbolic view. The sequences are named **U1**, **U2**, etc. The terms of each sequence are identified by number. For example, the first sequence is named **U1**. The first term of this sequence is **U1(1)**, followed by **U1(2)**, etc. $U1(n)$ is the general term and is used to enter the rule then used to get from the last term, which is $U1(n-1)$, to the current term. What should **U1(1)** and **U1(2)** be?

1. Use the sequence U1 to enter information about the US Total Population. What should **U1(1)** and **U1(2)** be?
2. When you enter an expression for $U1(n)$, use the common difference you used in the last activity. Your expression will look like this: **$U1(n) = U1(n-1) + \text{Common Difference}$** .
3. Press the **NUM** key to see a table for your sequence. How does the table shown on the calculator compare to the data in Table 5-1 above? If there are differences, how do you explain them?
4. Use the table of values for U1 to estimate the year in which the US population first reaches 400 million. Remember that each term in this sequence represents a 10-year census. Write down the two census projections you used to make your estimate, as well as the year of your estimate.

Exercise 2: Model the Hispanic Population

Examine the data in the second column of Table 5-1. Does the US Hispanic population appear to be as linear as the total US population? Let's use the sequence U2 to define a model for the US Hispanic population.

5. What should U2(1) and U2(2) be?
6. When you enter an expression for the data, will you use a common difference or a common ratio? How did you make that determination?
7. How did you come up with the common ratio or difference you used in your symbolic formula?
8. Create the table for your sequence on the calculator. How does the table shown on the calculator compare to the data in Table 5-1 given in this problem? If there are differences, how do you explain them?
9. Remember the year in which you estimated that the US population will first reach 400 million? What is the Hispanic population in that year? According to our models, what percent of the US population will be Hispanic in that year?
10. Use your table to estimate the year in which the Hispanic population first grows to be at least half of the total population of the US. Again, remember that your sequence jumps in 10-year steps. Write down the census projections you used to make your estimate, as well as the year of your estimate.

Exercise 3:

Let's now examine the data using the CensusData Aplet. Before you can answer any questions, you have to make sure you have the aplet on your calculator. Press the **APLET** key and scroll through your list of aplets to see if it is there. Once you have the aplet, make sure you can see the data for this problem in the **NUM** view. Alternately, you can just enter the data from Table 5-1 in **C1**, **C2**, and **C3** of the Statistics Aplet. Be sure to set the **1-Var/2-Var** menu key toggle to read **2-Var**! Use the same Plot Setup values used in the last activity as well.

In the Statistics Aplet, you can define up to five scatter plots. They are named, **S1**, **S2**, etc. Each one has a fit model that you can choose.

11. If you do not have the CensusData Aplet, press the **SYMB** key and set **S1** so it uses **C1** and **C3** as its data (year and US population). Then press **Shift SYMB** (Symbolic Setup) and choose a linear model for the **S1fit**.
12. Press **PLOT** to see the scatter plot. Then press the **MENU** key and the **FIT** key to see the fit as well as the data. Now you can trace along the line. Notice that the trace values are not ten years apart like the sequence values. Find the exact year in which the US population first reaches 400 million. How closely does this linear model agree with your sequence in **U1**? In particular, do they agree on the year in which the US population will first reach 400 million?
13. Now define **S2** to use columns **C1** and **C2** (year and Hispanic population). Make a choice for the model you will use as **S2fit**. Which model did you choose and why?
14. Explain any similarities and differences between the model found using statistics and the model you created in U2 of the Sequence Aplet. In particular, how closely do the two models agree on the year in which the Hispanic population first grows to be at least half of the total population of the US?

Exercise 4: Modeling a Ratio

In this exercise, rather than exploring the Hispanic population separate from the total US population, you will look at the ratio of the two populations.

15. To look at the US Hispanic population compared to the total US population, we will plot the year against the ratio of Hispanic to total US population. Turn off **S1** and **S2**. Now define **S3** to use **C1** and **C2/C3** (year and ratio of Hispanic to total). Change the vertical scale to reflect that the y-values are ratios. Then look at the graph and describe the pattern you see.
16. Use this model to predict when the Hispanic population will be at least 50% of the US population. You should be able to find the exact year from this model.
17. Does your answer to question 16 match your answer from the statistics model? Explain any difference.

Teacher Notes

This activity can be used to introduce linear and nonlinear functions as models. This activity extends the previous activity, which used a linear model for the US population from 1970 to 2050. In this activity, we add an exponential model for the Hispanic population of the US. The activity starts with recursively defined sequences using a common difference for the US population and a common ratio for the Hispanic population. Exploring the mathematics behind issues currently in the headlines can be a great way to generate student interest and motivation.

Teaching Standards

The following teaching standards can be found in this activity. These standards are general statements of what students should know and be able to do. The table below includes the general statement of each standard as well as its correlation to the Texas Essential Knowledge and Skills (TEKS) and the California Content Standards for Mathematics.

Represent patterns with tables, graphs, and algebraic equations, and make connections between those representations.		
CA	Grade 7, Algebra and Function	1.5: Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.
		3.4: Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.
	Algebra 1	6.0: Students graph a linear equation and compute the x- and y-intercepts.
TX	Grade 8	8.4: The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).
	Algebra 1	A.1(D): Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.
	Algebra 1	A.2(D): Collect and organize data, make and interpret scatter plots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.
	Algebra 1	A.5(A): Determine whether or not given situations can be represented by linear functions.

Answer Key

Exercise 1:

1. $U1(1) = 203,210,158$ and $U1(2) = 226,545,805$
2. The common differences used by students may vary. Figure 5-1 shows the final definition of the additive (linear) recursive model in U1. The common difference used here was 23,300,000.
3. Answers will vary. Figure 5-2 shows the first 6 terms. $U1(3)$ is reasonably close to the actual figure for 1990, while $U1(4)$ is quite a bit off due the greater change in the US population from 1990 to 2000.

SEQUENCE SYMBOLIC VIEW	
✓	$U1(1)=203210158$
✓	$U1(2)=226545805$
✓	$U1(N)=U1(N-1)+23300...$
	$U2(1)=$
	$U2(2)=$
EDIT	✓CHK
	SHOW EVAL

Figure 5-1

N	U1		
1	2.0321E8		
2	2.2655E8		
3	2.4985E8		
4	2.7315E8		
5	2.9645E8		
6	3.1975E8		
249845805			
ZOOM		BIG	DEFN

Figure 5-2

4. Figure 5-3 shows that the population reached 400 million between the 9th and 10th terms. Since the first term represents the population in 1970, our model projects that the population will reach 400 million between 2050 and 2060. Since the model is linear, students may start with the 2050 projection and add 2,330,000 per year to arrive at an estimate of 2054.

N	U1		
7	3.4305E8		
8	3.6635E8		
9	3.8965E8		
10	4.1295E8		
11	4.3625E8		
12	4.5955E8		
389645805			
ZOOM		BIG	DEFN

Figure 5-3

Exercise 2:

5. $U2(1) = 9,589,216$ and $U2(2) = 14,608,673$
6. Students should realize that they are not dealing with a common difference in this problem. While they may not be able to immediately recognize a common ratio, it should be clear that if it is not a common difference, they should be checking for a common ratio.
7. As with the common difference with the previous data, student answers will vary for the common ratio. They can use a guess-and-check method, choose only one of the ratios, or average the three. For the answers shown here, a common ratio of 1.54 is used.
8. Figure 5-4 illustrates the definition of the multiplicative (exponential) recursive model for the Hispanic population. Figure 5-5 illustrates the first few terms of the sequence, which fit the data reasonably well.

SEQUENCE SYMBOLIC VIEW	
✓	$U2(1)=9589216$
✓	$U2(2)=14608673$
✓	$U2(N)=U2(N-1)*1.54$
	$U3(1)=$
	$U3(2)=$
EDIT	✓CHK
	SHOW EVAL

Figure 5-4

N	U1	U2	
1	2.0321E8	9589216	
2	2.2655E8	14608673	
3	2.4985E8	22497356	
4	2.7315E8	34645929	
5	2.9645E8	53354790	
6	3.1975E8	82166285	
34645928.8868			
ZOOM		BIG	DEFN

Figure 5-5

9. Figure 5-6 illustrates the 9th and 10th terms of **U2**, indicating that, when the US population first reaches 400 million, the Hispanic population is projected to be over 300 million, or over 75% of the total population! On the other hand, notice that by 2060, the Hispanic population is projected to exceed the total US population! This is an excellent opportunity to discuss models, their strengths and shortcomings! Clearly, our models fall apart by 2060. Class

discussion could extend to defining effective domains for the models, or even revising the assumption of a linear model for the total US population.

N	U1	U2
7	3.4305E8	1.2654E8
8	3.6635E8	1.9487E8
9	3.8965E8	3.0009E8
10	4.1295E8	4.6214E8
11	4.3625E8	7.117E8
12	4.5955E8	1.096E9

300092964.53

ZOOM BIG DEFN

Figure 5-6

- Assuming that our models are still valid through 2040, Figure 5-6 illustrates that the Hispanic population first reaches the 50% mark between the 7th and 8th terms of the sequences, that is, between 2030 and 2040. Student estimates may vary, but will probably be between 2035 and 2039.

Exercise 3:

- The data (and the **2-Var** menu key) are shown in Figure 5-7. The definition of the scatter plot **S1** and its linear model are shown in Figure 5-8, while the graphs of the scatter plot and linear fit are shown in Figure 5-9.

n	C1	C2	C3	C4
1	1970	4589216	2.032E8	
2	1980	1.461E7	2.265E8	
3	1990	2.235E7	2.487E8	
4	2000	3.531E7	2.814E8	

1970

EDIT INS SORT BIG 2VAR STATS

Figure 5-7

STATISTICS SYMBOLIC VIEW	
✓S1:	C1 C3
✓Fit1:	m*X+b
S2:	
Fit2:	m*X+b

ENTER USER DEFINED FIT

EDIT ✓CHK X SHOW EVAL

Figure 5-8

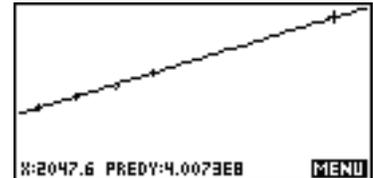


Figure 5-9

- The linear model best represents the arithmetic sequence in **U1**. However, the common difference used is not the same as the slope of our statistical model. The slope of the line is approximately 2,567,993/year, which corresponds to 25,679,931/decade - close to the common difference of 23,300,000 we used to define **U1**. Tracing along the linear model as in Figure 5-9 indicates the population first reaches 400 million on 2047. Our sequence model predicted this would occur after the 2050 census.
- We choose an exponential model because we used a common ratio to define our sequence for **U2**.

STATISTICS SYMBOLIC VIEW	
S1:	C1 C3
Fit1:	2567993.12*X-...
✓S2:	C1 C2
✓Fit2:	b*EXP(m*X)

ENTER USER DEFINED FIT

EDIT ✓CHK X SHOW EVAL

Figure 5-10

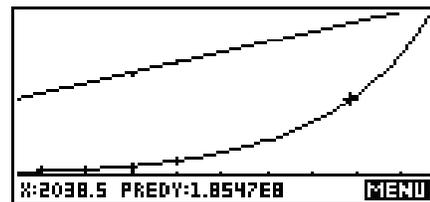


Figure 5-11

- An exponent model best represents a geometric sequence. However, the statistical fit equation is not the same as our sequence definition. The expression $e^{0.043356}$... found in the exponential fit is approximately 1.004 and represents the growth rate per year. Raising that to the 10th power to represent the growth rate for a decade gives us 1.5427..., which is close to the 1.54 common ration used in **U2**. Figure 5-11 illustrates the Hispanic population very close to half of the total population in 2038. This compares well with the estimates between 2035 and 2039 made with the sequence models.

Exercise 4:

In this exercise, rather than exploring the Hispanic population separate from the total US population, you will look at the ratio of the two populations.

- Figure 5-12 shows the definition of **S3**, while Figure 5-13 shows the Plot Setup. Figure 5-14 shows the graph, which appears exponential. Notice that you can define a scatter plot using expressions involving the various columns of data without creating new columns! This keeps the data set simpler and promotes exploration. Of course, you are welcome to create new columns to hold the ratio data if you want to do so.

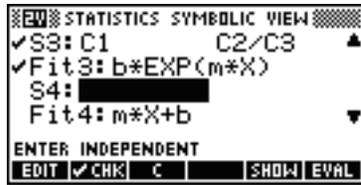


Figure 5-12



Figure 5-13

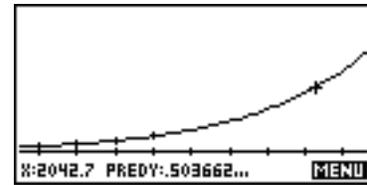


Figure 5-14

- According to the statistical model, the US Hispanic population will be at least 50% of the total population in 2042.
- Our sequence models predicted that the Hispanic population would first reach 50% of the total population between 2035 and 2039. Our exponential model predicts this will occur in 2042. Interestingly enough, this date matches exactly the current projections of the US Census Bureau.

6 Parent Functions



Author: Mark Howell

Introduction

This activity explores three similar and simple relationships between two variables, y and x :

- The sum of y and x is a constant
- The difference between y and x is a constant
- The quotient of y and x is a constant

These three relationships occur regularly in real-world applications. Here are a few simple examples:

1. Imagine a girl, Yvette, and her brother, Xavier, are going on a trip together. They have a case that holds 50 CDs. You would use the relationship $y + x = 50$ to describe the different ways Yvette and Xavier could contribute a certain number of CDs to fill the case.
2. The difference in Yvette's and Xavier's age is always constant.
3. If you measure the distance between any two points in both miles and kilometers and divide these measurements, the result is always a constant.

Can you think of other situations where each relationship would occur?

Exercise 1: $y + x = c$

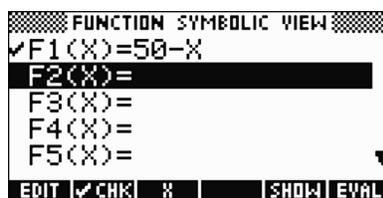
1. Each row in Table 6-1 represents one possible way Yvette and Xavier can contribute CDs to the case that holds 50. Fill in the missing number in each row so that the total number of CDs is 50. The first row has been done for you. In the bottom two rows, choose a value for either x or y . Then determine the other value so that the sum of x and y is 50.

Table 6-1 Solutions to $y + x = 50$

Solutions to $y + x = 50$	
y	x
23	27
39	
	49
50	
49	
	50
25	

Note that in every case, the value of y is always 50 minus the value of x . This suggests we could rewrite the equation $y + x = 50$ in an *equivalent* form as $y = 50 - x$.

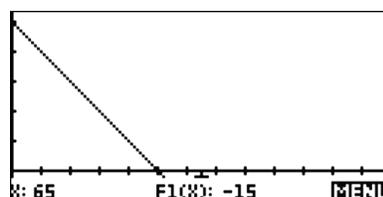
On the 39gs, an equation like $y = 50 - x$ is entered as **F1(X) = 50 - X**. Start the **Function** Aplet on the 39gs, and enter **F1(X) = 50 - X**.



Press **Plot Setup** and enter the values shown in the screen below:



Press the **PLOT** key to look at the graph of the equation $y = 50 - x$:

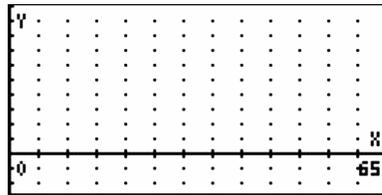


2. Press the left arrow key until the cursor is at the point corresponding to the situation where Xavier contributes 50 CDs and Yvette contributes 0. Mark the point on the graph above, and label it "A."
3. Press the left arrow key several more times. Describe the connection between the values you see each time you press left arrow and the contributions of Yvette and Xavier.
4. Press the left arrow key until the cursor corresponds to the situation where Yvette contributes 50 CDs and Xavier contributes 0. Mark the point on the graph above, and label it "B."

Now suppose that the CD case holds only 40 CDs.

5. What equation describes this situation?

6. Draw the graph of the equation on the axes below.



Extension 1:

Interpret the situation where the trace cursor is at the point where $y = -15$ and $x = 65$.

Extension 2:

Suppose Xavier's CDs are all in "double box sets," and use up twice the space as Yvette's CDs. Fill in a table, write an equation, and produce the graph for this setting. Now switch it around, and suppose Yvette's CDs are all in double box sets, while Xavier's are all singles. Again, fill a table, write an equation, and produce a graph for this setting.

Exercise 2: $y - x = c$

Yvette is 4 years older than Xavier. If we use y to represent Yvette's age and x to represent Xavier's age, then the difference, $y - x$, is always a constant, 4.

- Each row in Table 6-2 represents one possible pair of ages for Yvette and Xavier. Fill in the missing number in each row. The first row has been done for you. At the bottom, there is room for you to fill in values in *both* columns. Choose your own value for either x or y , then determine the other value so that the equation is satisfied.

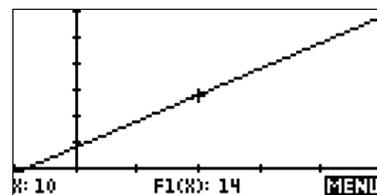
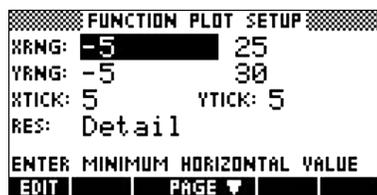
Table 6-2 Solutions to $y - x = 4$

Solutions to $y - x = 4$	
y	x
7	3
9	
	12
17	
21	
	29
40	

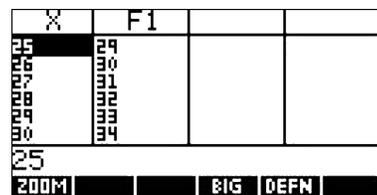
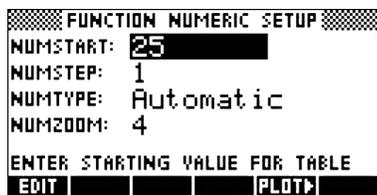
Note that in every case, the value of y is always 4 more than the value of x . This suggests we could rewrite the equation $y - x = 4$ in an *equivalent* form as $y = 4 + x$.

On the 39gs, an equation like $y = 4 + x$ is entered as **F1(X) = 4 + X** into the Symbolic view of the **FUNCTION** Aplet.

Once the function is entered, change the **Plot Setup** as shown. Then press the **PLOT** key to look at the graph of the equation $y = 4 + x$:

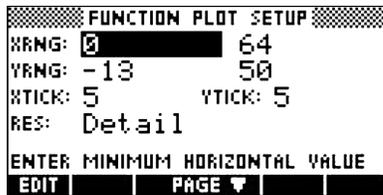


Now press **NUM** Setup (**SHIFT-NUM**), and enter the values shown for **NUMSTART** and **NUMSTEP**. Then press **NUM**, and you should see the table of values on the right.

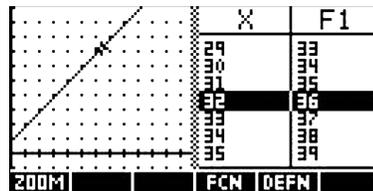


- Press **0**, then press **Enter**, and you'll see the table adjust. This row in the table corresponds to the year that Xavier was born, ($x=0$). How old was Yvette when Xavier was born?
- On the graph on the previous page, mark and label with a "B" the point that corresponds to the situation when Xavier was born.

The 39gs can show you a special view with the graph and table linked together on the same screen. First, change the **Plot Setup** as shown below.



Now press the **VIEWS** key, and select **Plot-Table**.



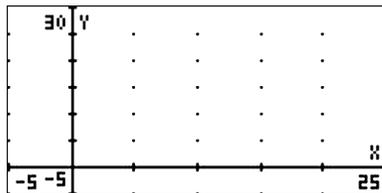
4. Press the right arrow key several times, watching how the values of x and y change as you do so. For each year that Xavier gets older, how much does Yvette's age increase?

5. Press the left arrow key several times, watching how the values of x and y change as you do so. What is the situation when x is negative and y is positive?

Now suppose that Yolanda is 6 years older than Xavier.

6. What equation describes this situation?

7. Draw the graph of this equation on the axes below. A grid is provided to help you.



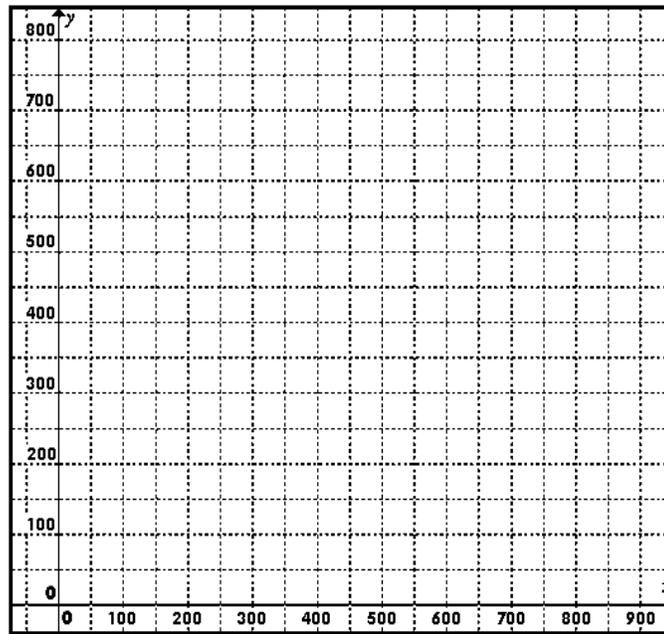
Exercise 3: $y/x = c$

Each row in Table 6-3 contains the distance between Austin and another Texas city, measured in both kilometers and miles.

Table 6-3 Driving Distances from Austin

Driving Distances from Austin		
City	$x = \text{kilometers}$	$y = \text{miles}$
San Antonio	127.8	79.4
Dallas	314.8	195.6
Corpus Christi	349.4	217.1
Houston	261.5	162.5
El Paso	929.7	577.7
Huntsville	270.4	168
Laredo	376.4	233.9

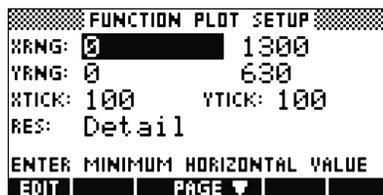
Make a graph of $y = \text{distance from Austin in miles}$ versus $x = \text{distance from Austin in kilometers}$.



1. What geometric shape is the graph?
2. Is the point $x = 0, y = 0$ on the graph? Explain why or why not.
3. Use your calculator to compute the ratio y/x for each row in the table. What ratio did you get?
4. Your calculation in question 3 suggests that the equation $y/x = c$ describes the relationship between $y = \text{distance from Austin in miles}$ versus $x = \text{distance from Austin in kilometers}$. What is the value of c ?

We can rewrite the equation $y/x = 0.62$ in an equivalent form as $y = 0.62x$. On the 39gs, an equation like $y = 0.62x$ is entered as **F1(X) = 0.62*X**. Edit the definition for **F1**, changing it to **F1(X) = 0.62*X**.

Enter the **PLOT SETUP** shown below. Then graph the equation.



Note that when you trace on the graph in this view, the value of X changes by 10 kilometers every time you press a right or left arrow key.

5. Jody lives 250 kilometers from Austin. How many miles is Jody from Austin?

6. Can you use trace to find out how many miles the distance 512 kilometers is? Explain.

You can determine the distance in miles given any distance in kilometers using the calculator's **GOTO** function. On the graph screen, press the menu key, then press the **GOTO** key. Enter **512**, a distance in kilometers. Press **OK** or **ENTER**.

7. How many miles away is a city that is 512 kilometers from Austin?

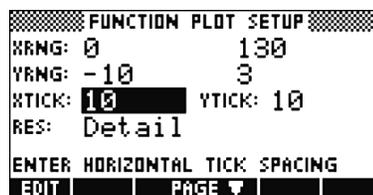
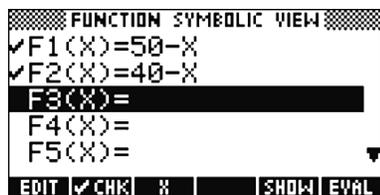
8. Shown below is a small map of Texas, with a scale so that 1.5 inches is equivalent to about 240 miles. On the map, show all the points that are 240 miles from Austin.



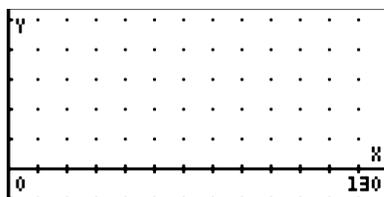
9. On the calculator, **TRACE** until you get to the point with a distance close to 240 miles from Austin. Remember, the y-coordinate measures miles! Approximately how many kilometers from Austin is this?
10. Now define $F_2(X) = 240$, and find the intersection between $F_1(X)$ and $F_2(X)$. You do this by pressing **MENU**, and then selecting **Intersection** from the **FCN** menu. You'll be prompted for the intersecting functions, choose $F_1(X)$ and $F_2(X)$. Exactly how many kilometers away is a point that is 240 miles from Austin?
11. Suppose you are on a highway leading away from Austin. For each kilometer of increase in your distance from Austin, how many more miles are you from Austin?

Exercise 4: Consolidation

The previous three exercises had one main feature in common: every graph you produced was linear. *Linearity* is a key idea in mathematics. It is studied in various forms in all of high school mathematics, and beyond. In this activity, you will explore connections between linear equations and the graphs and tables that they produce. In Exercise 1, you examined the graphs of the equations $y = 50 - x$ and $y = 40 - x$. Let's examine these again. Define $F1(X) = 50 - X$ and $F2(X) = 40 - X$, and enter the **PLOT SETUP** shown:



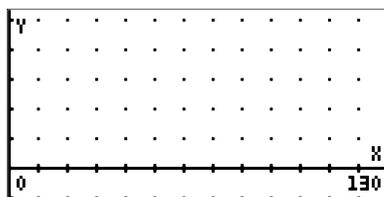
1. Draw the graphs on the screen below. Then without using your calculator, draw the graphs of $y = 30 - x$ and $y = 20 - x$ on the axes below. Once you've finished, check your graphs by appropriately defining **F3(X)** and **F4(X)** and drawing the graphs on the calculator.



2. You've graphed the equations $y = 50 - x$, $y = 40 - x$, $y = 30 - x$, and $y = 20 - x$. In the equation $y = c - x$, what is the graphical significance of the value of c ?

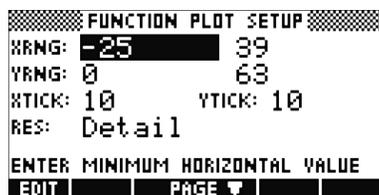
If you completed Extension 2 in Exercise 1, you examined the graph of the equation $y = 50 - 2x$. Let's examine it again. Leave $F1(X) = 50 - X$, redefine $F2(X) = 50 - 2X$, and clear out the definitions of **F3(X)** and **F4(X)**.

3. Draw the graphs on the screen below. Then without using your calculator, draw the graph of $y = 50 - 4x$. Once you've finished, check your graphs by appropriately defining **F3(X)** and drawing the graph on the calculator.



4. You've graphed the equations $y = 50 - x$, $y = 50 - 2x$, and $y = 50 - 4x$. In the equation $y = 50 - mx$, what is the graphical significance of the value of m ?

Now we'll take a look at the numeric significance of the value of m . Change your **PLOT SETUP** as shown, and select **Plot-Table** from the **VIEW** menu:



5. Look at the graph, and the table of values for **F1(X)**. When X increases by 1, what is the change in **F1(X)**?

Press the down arrow key, and the cursor will move to the graph of **F2(X)**.

6. Look at the graph, and the table of values for **F2(X)**. When X increases by 1, what is the change in **F2(X)**?

Press the down arrow key one more time, and the cursor will move to the graph of **F3(X)**.

7. Look at the graph, and the table of values for **F3(X)**. When X increases by 1, what is the change in **F3(X)**?

8. Summarize your findings about the connection between the value of m and the change in the value of y when x increases by 1.

Extension 1:

Revisit Exercise 2, using older brother younger sister = 4, 6, 12. Compare the graphs.

Extension 2:

Revisit Exercise 3, in the context of currency conversion, x = Dollars, y = Euros. Use various historical conversion rates. Draw the graph for each, and interpret the rate as the slope of the line.

Teacher Notes

These activities can be used to introduce students to the notions of parameters and parent functions. These activities all concentrate on the parent functions $y = cx$ and $y = x + c$, although they are introduced in the form $y + x = c$, $y - x = c$, and $y/x = c$. The last form, $y \cdot x = c$, is treated in the another activity, *The Pressure's On*. By concentrating on equations in which a single operation on two variables results in a constant, students learn to look for the role of the constant in the graph of the function.

Students are required to use ratios, tables and symbols to represent and describe proportional situations, and analyze patterns and examples using mathematical properties and relationships. These activities also introduce students to equations with linear graphs. The first activity begins with an investigation of pairs of numbers whose sum is 50. Students discover that the graph of such an equation is a line with a y -intercept of 1 and slope -1 . The terms "slope" and " y -intercept," however, are not used in the exercise. The emphasis is on recognizing linearity, and the consequent rate connection between the two variables. In this case, for every additional CD that Xavier contributes, Yvette contributes one CD fewer.

Extension 1 is really just one additional question. It asks students to interpret a negative value for y .

Extension 2 offers a nice context in which to explore this rate connection further, depending on the grade level of the students and needs of the teacher. If Xavier only contributes "double box sets," then for each additional CD set that he contributes, Yvette contributes two CDs fewer. The interpretation of the last scenario, in which Yvette contributes doubles and Xavier singles, is the most difficult. For every 2 additional CDs the Xavier contributes, Yvette contributes one fewer.

The second exercise is similar to the first. Here, the difference between two quantities is held constant, using the context of the ages of Yvette, Xavier, and Yolanda.

The third exercise, though it leads to a linear graph, is nonetheless quite different cognitively. It involves a simple direct variation between distance measured in miles and in kilometers. Students graph ordered pairs representing distances in kilometers and miles between Texas cities. They discover a simple relationship: the ratio of the distances is constant.

The final exercise leverages concepts from the first three, bringing the results together. The focus here is on recognizing the significance of parameters in linear equations of the forms $y = c - x$, $y = c + x$, and $y = mx$.

TEKS

The following TEKS can be found in this activity set. The TEKS are listed in grade-level order, not the order in which they appear in the activities. Also, some of the TEKS statements are abbreviated or paraphrased here.

The student is expected to:

- 6.3(A) use ratios to describe proportional situations
- 6.4(A) use tables and symbols to represent and describe proportional and other relationships
- 6.13(A) make conjectures from patterns or sets of examples and nonexamples
- 6.13(B) validate his/her conclusions using mathematical properties and relationships

Table 6-4 correlates the Exercises to the TEKS so that the teacher can fine tune a lesson to meet the needs of the class.

Table 6-4 TEKS Covered, by Activity

Activity	TEKS Covered
Exercise 1	6.4A, 7.4C, 7.5B, 8.4, 8.5A
Extension 2	7.5B
Exercise 2	6.4A, 7.4C, 7.5B, 8.4, 8.5A
Exercise 3	6.3A, 6.4A, 7.3B, 7.4A, 7.4B, 8.3B, 8.4, 8.5A
Exercise 4	6.13A, 6.13B, 7.4B, 8.4, 8.5A, 8.5B

Answer Key

Exercise 1

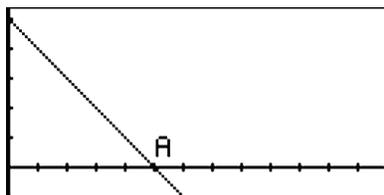
1.

Table 6-5 Solutions to $y + x = 50$

Y	x
23	27
39	11
1	49
50	0
49	1
0	50
25	25
21	29
-10	60

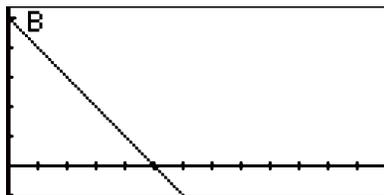
Answers will vary for the last two rows.

2.



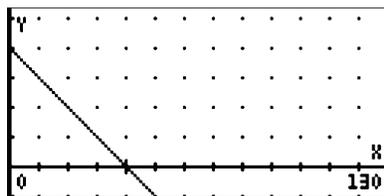
3. Every time you press left arrow key, the contribution of Xavier goes down by 1 and the contribution of Yvette goes up by 1.

4.



5. $y = 40 - x$

6.



Extension 1

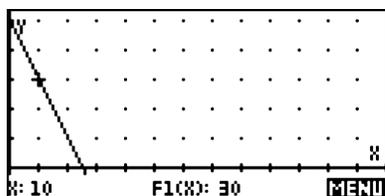
If $y = -15$ and $x = 65$, this would mean that Xavier contributed 65 CDs and Yvette took 15 out of the case.

Extension 2

The equation for the situation in which each CD Xavier contributes is a "double box set" is $y + 2x = 50$.

Solutions to $y + 2x = 50$

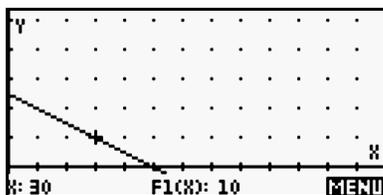
y	x
10	20
40	5
2	24
48	1
50	0
0	25



If Yvette's are all "doubles," the equation is $2y = 50 - x$ or $y = \frac{50 - x}{2}$.

 Solutions to $2y + x = 50$

y	x
20	10
5	40
24	2
1	48
0	50
25	0



Exercise 2

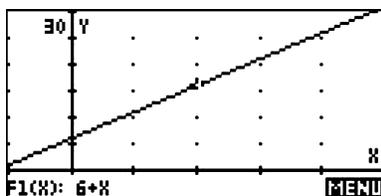
1.

Table 6-6 Solutions to $y - x = 4$

y	x
7	3
9	5
16	12
17	13
21	17
33	29
40	36

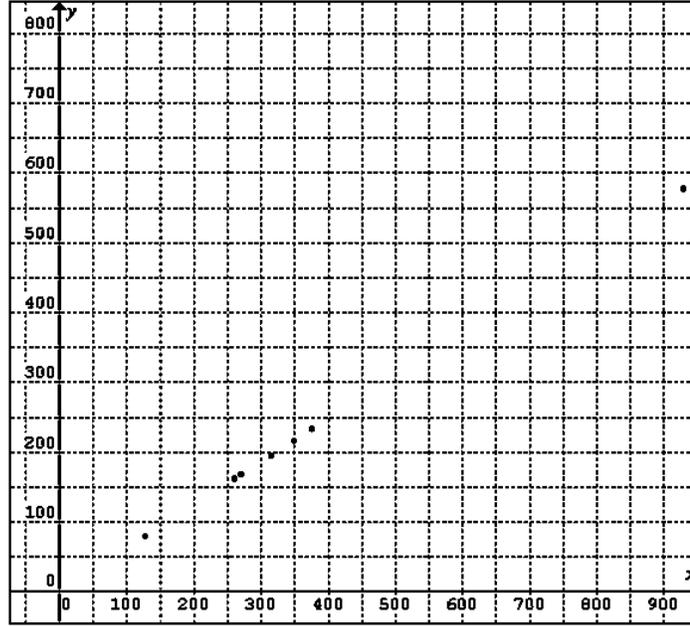
Answers will vary for the last two rows.

- 4
- Point B is at the y-intercept of the line, (0, 4).
- For each year that Xavier gets older, Yvette's age increases by one year
- These correspond to years after Yolanda was born, but before Xavier was born.
- $y = 6 + x$
-

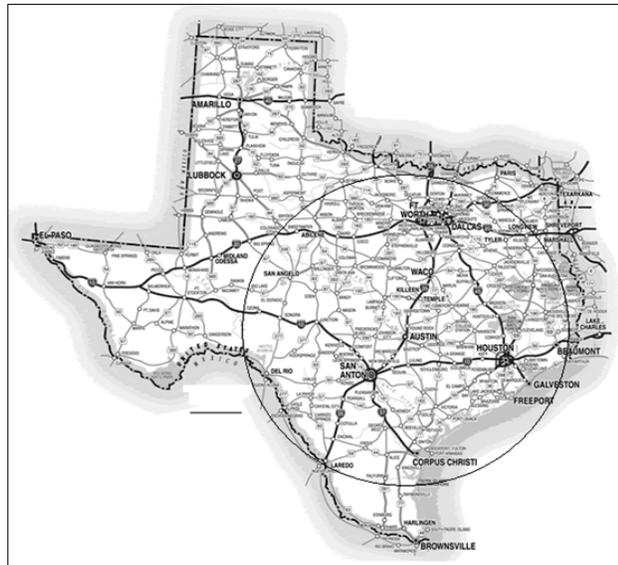


Exercise 3

- The graph is linear.



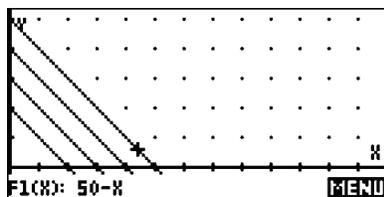
- Yes. Austin is 0 miles and 0 kilometers from itself.
- Approximately 0.62
- Approximately 0.62
- 155 miles
- 317.44 miles
- Students should draw a circle on the map centered at Austin with a radius of about 1.5 inches.



- About 390 kilometers
- 387.1 kilometers
- 0.62

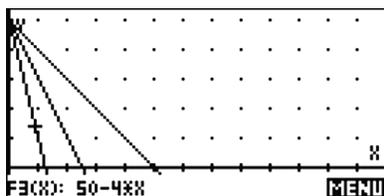
Exercise 4

1.



2. The value of c corresponds to the y -coordinate of the point where the graph intersects the y -axis.

3.



4. The value of m corresponds to the steepness of the graph. When m is larger, the graph is steeper.

5. When x increases by 1, y decreases by 1.

6. When x increases by 1, y decreases by 2.

7. When x increases by 1, y decreases by 4.

8. In the equation $y = 50 - mx$, when the value of x increases by 1, the value of y decreases by m .

Web Resources

http://www.tripinfo.com/images/state_maps/TX.gif

7 Zeros of Quadratics and their Linear Factors



Author: Michael Grasse

Introduction

Much can be learned about the structure of polynomials by looking at their graphs along with the graphs of their linear factors. What can we learn about the function of $y = (x - 1)(x - 5) = x^2 - 6x + 5$ by looking at the graph of $y = x - 1$ and $y = x - 5$ separately? More specifically, what can we learn about the zeros of the function $y = x^2 - 6x + 5$ by looking at the zeros of the much simpler $y = x - 1$ and $y = x - 5$?

This activity requires students to create their own sign graphs for polynomial expressions. They also analyze the functions using their own graphs to determine the signs for each linear factor, the sign for the product of the linear factors, and the vertical lines at the zeros.

Exercise 1:

Start the **Function** Aplet and define **F1(X) = X - 1** and **F2(X) = X - 5**. Press **SHIFT PLOT** and **SHIFT DEL** to set the standard plotting window. Press **PLOT**, and you should see what is pictured in the figure below.

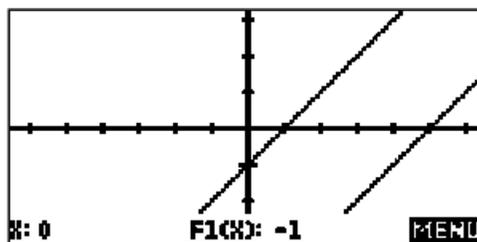


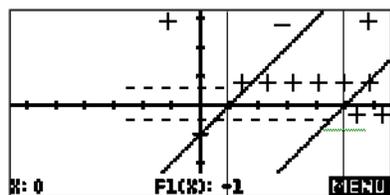
Figure 7-1

Is it possible to tell how many zeros $y = x^2 - 6x + 5$ has by just looking at the graphs of the linear factors?

Can you tell where the zeros are from looking at the graph? For what values of x is $y = x^2 - 6x + 5$ positive?

Negative? Look at the sign chart below to help answer these questions. You will have to make your own sign chart for the other examples. In the chart below, the sign (+ or -) of $y = x - 1$ is written directly above the x -axis, and the sign of $y = x - 5$ is written directly below the x -axis. The sign of the product,

$y = x^2 - 6x + 5$, is written at the top of the graph, along with two vertical lines that mark where the product changes signs.



$$F3(X) = F1(X) \cdot F2(X)$$

$$F1(X) = X - 1$$

$$F2(X) = X - 5$$

Figure 7-2 Sign chart

Look at the graph and sign chart to answer the following questions:

1. What are the zeros (the values of x that give a y -value of zero) of $y = x - 1$ and $y = x - 5$?
2. What are the zeros of the product, $y = (x - 1)(x - 5) = x^2 - 6x + 5$?
3. Between what two x -values is the function $y = x^2 - 6x + 5$ negative?
4. What are the signs of $y = x - 1$ and $y = x - 5$ in the interval where $y = x^2 - 6x + 5$ is negative?

Of course, the HP 39gs can also graph quadratics. Press **SYMB** and define **F3(X) = F1(X)*F2(X)**. We could have just defined **F3(x) = X^2 - 6X + 5**, but we will see later that what we actually did produces the same result and is much more flexible.

5. How does the parabola for $y = x^2 - 6x + 5$ match the sign chart?

Exercise 2:

What would happen if we took the opposite of each factor? Press **SYMB** and define **F1(X) = -(X - 1)** and **F2(X) = -(X - 5)**. The parentheses are important!

Make sure that you uncheck **F3(X)**. You do not want to see the graph of the parabola yet. Notice that **F3(X)** will give us the product of the linear factors every time. The standard plotting window should work fine, so just press **PLOT** when the functions are typed in.

Make a sign chart on the axis provided.

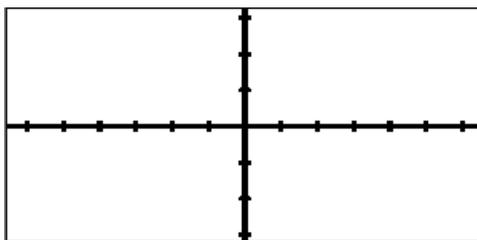


Figure 7-3

Looking at your sign chart:

6. What are the zeros of $y = -(x - 1)$ and $y = -(x - 5)$?
7. Multiply $-(x - 1) \cdot -(x - 5)$ by hand. Show your work.

8. What are the zeros of the product, $y = -(x - 1) \bullet -(x - 5)$?

9. How will this parabola graph be different than the graph of the parabola in Exercise 1?

Graph **F3(X)** if you are unsure how to answer question 9.

Exercise 3:

Press **SYMB** and define **F1(X) = (X - 1)** and **F2(X) = (3 - X)**. Make sure that you uncheck **F3(X)**. You do not want to see the graph of the parabola yet. Press **PLOT** when the functions are typed in. Make a sign chart on the axis provided.

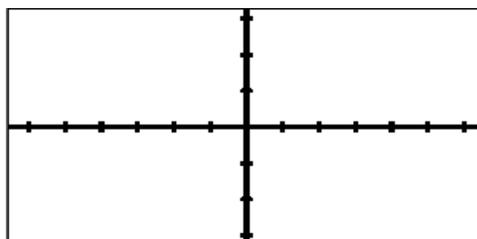


Figure 7-4

Looking at your sign chart:

10. What are the zeros of $y = (x - 1)(3 - x)$?

The *vertex* of a parabola is the highest (or lowest) point on the graph. If the parabola opens upward, the vertex is the lowest point of the graph. If the parabola opens downward, the vertex is the highest point of the graph. Now press **SYMB** and check **F3(X)** and graph the parabola along with the linear factors.

11. Which way does this parabola open?

12. What are the coordinates of the vertex?

13. Do you notice any relationship between the vertex and the graphs of the linear factors?

14. Is the relationship that you found in question 13 just a coincidence? Change one of the numbers in **F1(X)** or **F2(X)** and explain what you see.

Exercise 4:

What linear factors will produce the reflection of the polynomial in Example 3 in the x-axis? In other words, how do you change the factors $y = x - 1$ and $y = 3 - x$ to flip the parabola over the x-axis?

15. Does reflecting a polynomial in the x-axis change the zeros of the polynomial?
16. What two linear factors will reflect the graph of $y = (x - 1)(3 - x)$ over the x-axis? Experiment with linear factors until you find the two that work. Write them down. Can you find others?

Exercise 5:

What about $y = (x - 3)(x - 3) = x^2 - 6x + 9$?

Define **F1(X) = X - 3** and **F2(X) = X - 3**. Make sure that you uncheck **F3(X)**. You do not want to see the graph of the parabola yet. The standard plotting window should work fine so just press **PLOT**. It looks like only one line has been graphed, but there are actually two graphs of an identical line – two of the very same line!

Is it possible to tell how many zeros $y = x^2 - 6x + 9$ has by just looking at the graphs of the linear factors? Can you tell where the zeros are by looking at the graph? Make a sign chart.

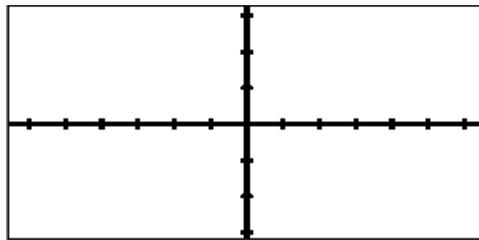


Figure 7-5

17. For what values of x is $y = x^2 - 6x + 9$ positive? Negative?
18. Propose a rule for all quadratic functions of the form $y = (x - a)(x - a)$?

Teacher Notes

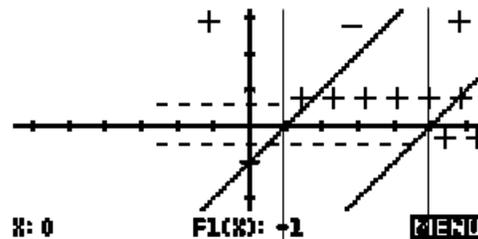
This activity requires students to create their own sign graphs for polynomial expressions and analyze the functions using their own graphs to determine the signs for each linear factor, the signs for the product of the linear factors, and the vertical lines at the zeros.

Sign charts are used in mathematics classes from beginning algebra through calculus. The level of detail required here may seem daunting to students, but good habits acquired now will pay dividends for their conceptual understanding later.

The first sign chart is made for the student. All of the components (the signs for each linear factor, the signs for the product of the linear factors and the vertical lines at the zeros) should be present in subsequent student-made sign charts.

Answer Key

Questions 1– 5 refer to the sign chart below:

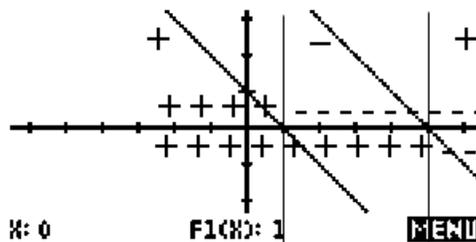


1. The zeros are $x = 1$ and $x = 5$ respectively.
2. The zeros of $y = x^2 - 6x + 5$ are $x = 1$ and $x = 5$.

NOTE: the zero product property is responsible for ensuring that the zeros of the quadratic polynomial are the same as the zeros of the linear factors. This fact cannot be overemphasized.

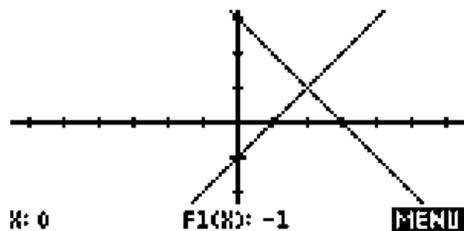
3. $y = x^2 - 6x + 5$ is negative between $x = 1$ and $x = 5$.
4. $x - 1 > 0$ and $x - 5 < 0$ when x is strictly between 1 and 5.
5. The y -value (as read off the parabola) is positive, negative and zero precisely where the sign chart predicts.

Questions 6 – 9 refer to the sign chart below:

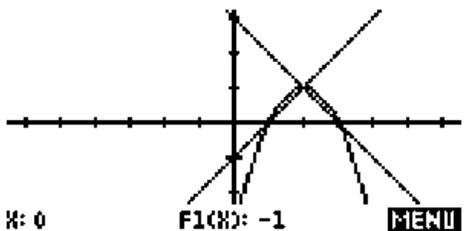


6. The zeros are $x = 1$ and $x = 5$ respectively.
7. $-(x-1) \cdot -(x-5) = (x-1)(x-5) = x^2 - 6x + 5$
8. The zeros are $x = 1$ and $x = 5$.
9. Even though the sign chart is different, this parabola is the same as in Example 1.

Questions 10 – 14 refer to the sign chart below:

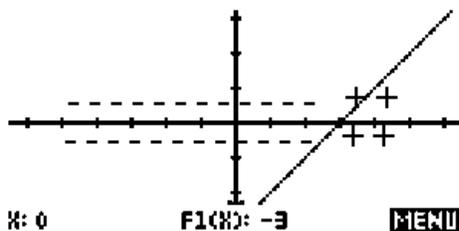


10. The zeros are $x = 1$ and $x = 3$.
11. The parabola opens downward.
12. The vertex is at $(2, 1)$.
13. The vertex is at the same point as the intersection of the graphs of the linear factors.



14. This is a coincidence as any change in linear factors will probably yield a counter example.
15. No, the zeros stay the same.
16. Multiplying one of the factors by -1 will reflect the graph over the y -axis.

Questions 17 – 18 refer to the sign chart below:



17. $y = x^2 - 6x + 9$ is positive for all real number inputs.
18. All quadratic functions of the form $y = (x - a)(x - a)$ have only one zero (at $x = a$) and no sign changes.

8 Probability and Dice



Author: Michael Grasse

Introduction

This activity requires students to develop a best strategy for winning at a game of chance. It also introduces the concepts of theoretical and experimental probabilities. Many board games use the sum of two dice to add an element of chance. Let's play a game based on an old board game called, *Snake Eyes*. The rules of this two-player game are:

- Each player gets 18 chips and one game board.
- Each player places chips on the numbers of the board any way he or she wishes (numbers can have more than one chip placed on them), but once the dice rolling starts, chips cannot be moved.
- The first player rolls two dice and adds the numbers showing on the dice. If the first player has a chip, or chips, on that number on his or her board, he or she may take one chip off of that number. If there is no chip on that number, no chips are removed.
- The second player then gets a turn.
- Players keep taking turns. The first player to remove all of the chips from his or her board wins.

Try the game, then come back and answer some questions.

1. Is there a number on the game board that you definitely do NOT want to put any chips on? Why not?

2. Do you think there is a roll that happens most often? If so, what roll do you think is the most common?

Experimenting to Find the Best Strategy

Let's investigate to see if there is a best strategy for placing chips on the board. One way to do this is to play the game backwards. Start with an empty game board. Every time you roll the dice, place a chip on the number that is equal to the sum of the dice. This is not an actual game, but it might help us see a pattern.

3. Draw what your game board looks like when you have placed all your chips.

1	2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	---	----	----	----

The drawing you just made is a type of a graph known as a *histogram*. A histogram is a display of statistical information that shows the frequency of data items. The higher the bar of a histogram, the more frequently that data item occurs.

Using the HP 39/40gs to Roll the Dice

Rolling dice is fun, but it is also time consuming. The Aplet called **DICE** uses the HP 39/40gs to simulate rolling two dice and finding the sum. It stores the results in a couple of useful ways. Let's try it.

Start the **DICE** Aplet. Press the **VIEWS** key and press **ENTER**. Select the number of dice rolls you would like to simulate (the default is 36). After the HP 39/40gs "rolls the dice" the requested number of times, note that **C1** contains the dice sums from 1 to 12, and **C2** contains the frequency of that roll. The sum of all the entries in **C2** should add up to the total number of rolls.

4. What is the most common roll in the calculator simulation?

5. What is the second most common roll?

Press **PLOT** to see a histogram of this simulation.

Press **VIEWS** to start over and do one more simulation.

6. Did the most common and second most common dice rolls change from the first simulation? If so, what are they for this second simulation?

Computing Probabilities

Instead of just counting the frequency of each dice roll, it may be useful to compute the *probability* of each dice roll. The probability of a particular dice roll is given by:

$$\text{Probability of a particular dice roll} = \frac{\text{Frequency of a particular dice roll}}{\text{Total number of dice rolls}}$$

This allows us to compare results from different simulations even if the total number of dice rolls is different. The HP 39/40gs makes this easy. On the **HOME** screen (but still in the **DICE** Aplet), type **C2**, divide that by the total number of dice rolls, press the **STO** menu key and type **C3**. When you press **NUM**, you will see each of the probabilities in **C3**.

7. What dice roll has the greatest probability? What is that probability?

8. What dice roll has the least probability? What is that probability?

Experimental Versus Theoretical Probabilities

The probabilities computed so far are *experimental probabilities*, because they are based on experimental data. We can also compute the *theoretical probability* for each dice roll by counting the total number of ways that two dice may be rolled and then counting the number of ways each sum may be rolled. Examine Table 8-1 below. The first column represents all the ways the first die can be rolled, while the first row represents all the ways the second die can be rolled.

9. Fill in the blank parts of the table with sum of the first and second die. A few of them have been done for you.

Table 8-1: Sum of Two Dice

	1	2	3	4	5	6
1						
2		4				
3						
4						10
5				9		
6						

10. How many different possibilities are there when rolling two dice?
11. One way to answer the above question is to count the number of spaces in the table that contain a sum. Is there a quicker way? What is it?

Now look at the table below. The first row contains all of the sums possible when rolling two dice. A list of all the possible outcomes for an event is called the *sample space*.

12. Fill in the second row with the number of ways that each sum can occur. In other words, use Table 8-1 to count the number of ways you can roll a sum of two, then count the number of ways you can roll a sum of three, and so on.

Possible Sums	2	3	4	5	6	7	8	9	10	11	12
Number of Ways											

Remember, to find the probability, you have to divide the frequency of a particular roll by the total number of possible dice rolls (the answer to question 10).

13. What is the theoretical probability of rolling a dice sum of 5?

14. What is the theoretical probability of rolling a dice sum of 7?

15. Fill out the following table of theoretical probabilities.

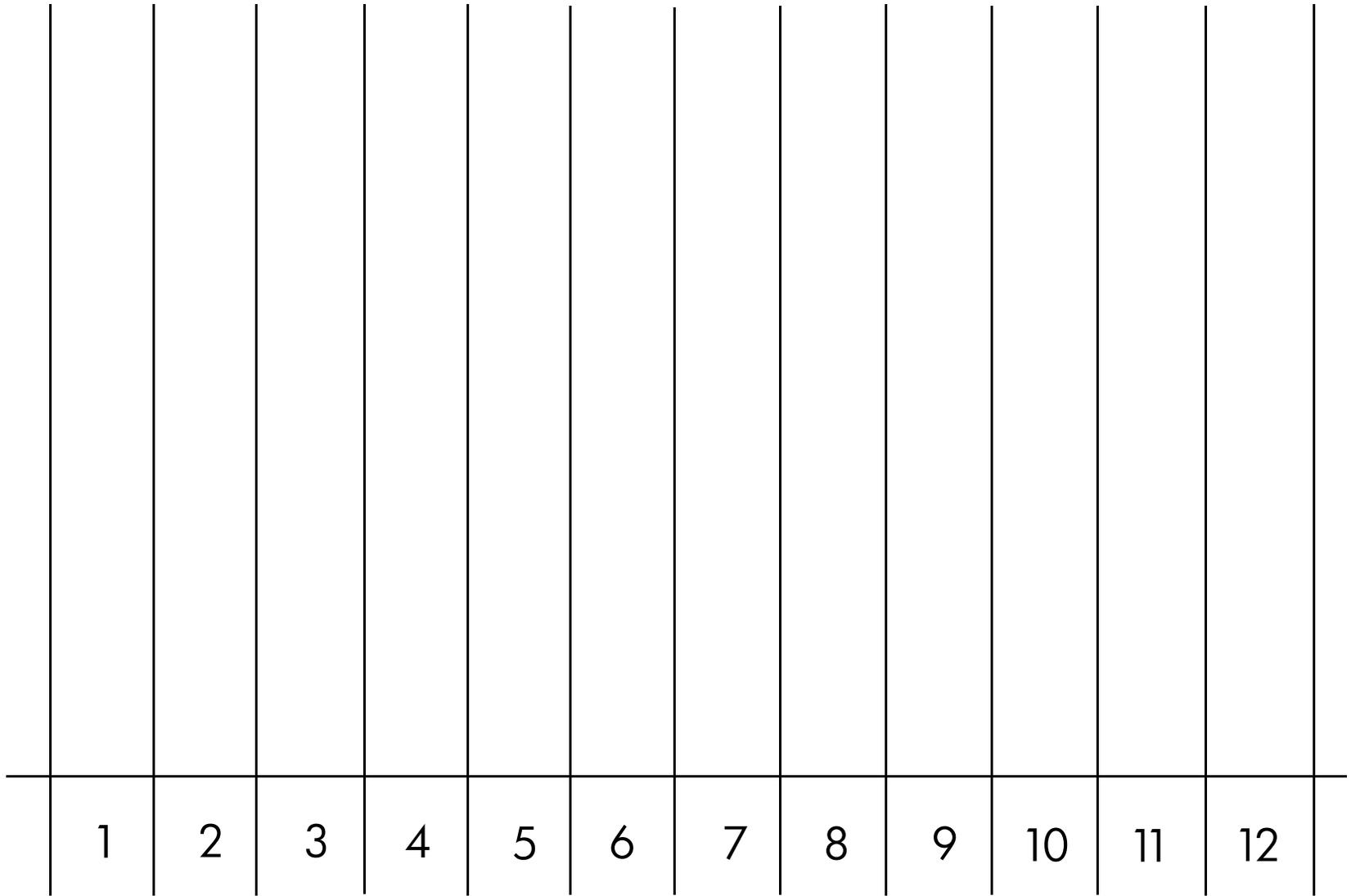
Possible Sums	2	3	4	5	6	7	8	9	10	11	12
Probability											

16. Do you see a pattern in probabilities of all the dice sums? What dice sum has the same probability as rolling a sum of 4?

17. Suppose you are playing Snake Eyes with 36 chips. What is theoretically the best way to place your chips? Fill in the table below:

Number of Chips												
Sum	1	2	3	4	5	6	7	8	9	10	11	12

18. Are you guaranteed to win if you set up your board this way? Why or why not?



9 The Coin Toss Game: A Student Investigation



Author: G.T. Springer

Introduction

The Coin Toss game is a familiar sight at carnivals and fairs across the United States. In this game, the player tosses a coin or disk onto a grid of squares. If the coin lands entirely within a square, touching no lines on the grid, then the player wins a prize.



Figure 9-1 The Coin Toss Game

In this investigation, you are assigned to work on a coin toss game for your school's club fair. The Math Club has a grid of 10 cm squares, but they can't find their disks. Your job is to help them figure out how big the disks should be. Larger disks will make the game harder to win and generate more money for the Math Club, but people won't want to play if the disks seem too big to them. Smaller disks will make the game easier to win but will reduce profits for the Math Club. It's up to you to help them decide!

1. Suppose that the prizes given out to winners of the coin toss game cost the Math Club \$1. Also suppose that the game costs \$0.25 to play. Discuss with your class what the probability of winning needs to be in order for the Math Club to make money hosting this game. Your probability should be expressed as a fraction between 0 (nobody ever wins) and 1 (everybody always wins). For example, a probability of $\frac{1}{4}$ or 0.25 means 1 in 4 tosses wins. A probability of 0.25 means the Math Club just breaks even because, on average, for every four players the Math Club makes \$1, but there is one winner who costs them \$1. Write what you think is the best value or range for the probability of winning the game. Your probability must allow the club to make money!
2. Once the class has decided on the best theoretical probability for your game, you will need to figure out the diameter of the disk that matches your chosen probability. To help you get some experimental evidence, your teacher will send you an HP 39gs Aplet called *Coin Toss 1*. This aplet will let you choose a diameter for your disk and simulate playing the game any number of times that you like. You can run as many trials as you like, each with the same number of tosses. The aplet will show you each toss, how many wins you have so far, and what the experimental probability of your disk is at that point. Your teacher will divide the class into teams, each with a diameter to explore.

- A. On your HP 39gs, go to the Aplet Library (press the **APLET** key), select the **Coin Toss 1** Aplet, and press the **START** menu key (Figure 9-2). You will be asked for the number of tosses in your trial and the diameter of your coin. The number of tosses must be 1 or greater (Figure 9-3) and the diameter of your coin must be greater than 0 and less than 10. Once you make your choices, you will then see the main menu, as shown in Figure 9-4. You can return to this menu at any time by pressing the **VIEWS** key.



Figure 9-2 Start Coin Toss

- B. To run your simulation, choose the **Toss Coins** option from the main menu. You can choose to run the trial manually (press any key to do each toss one at a time) or automatically. Once you start your trial, your display will look similar to Figure 9-5, showing you the result of your first toss.



Figure 9-3 Select # of tosses

- C. At the end of your trial, you will be asked if you wish to save the data from your trial. Select **YES** from the menu. The probability of winning from your simulation will be saved, and you can run another simulation.



Figure 9-4 Start Tossing!

- D. Once you have run several trials, you can look at the histogram or box-and-whisker plot of the probabilities of all your trials. Just make your selection from the main menu.

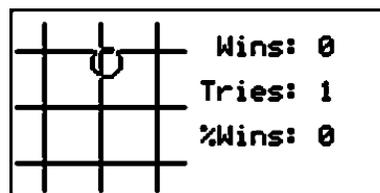


Figure 9-5 Results of 1st toss

- E. Figure 9-6 shows a sample of what the histogram could look like. From this histogram, you should be able to estimate the range for the theoretical probability associated with a coin of the size your group has chosen. Write this value or range below.

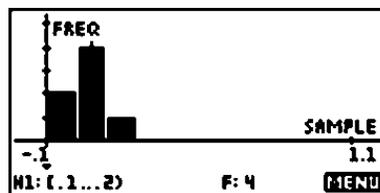


Figure 9-6 Graph of 7 trials

3. Post your results on the table your teacher has written on the blackboard. When all groups have added their data to the table, use the table to discuss what diameter coin is most likely to have the probability you chose for your game. Write this diameter below.

Now we'll look at how to determine the theoretical probability. The coin in Figure 9-7 is just touching the edge of the square.

4. Use a ruler to measure the dimensions of the square. Do you agree that it is 10cm x 10cm? Now measure the diameter of the coin and record its size below.

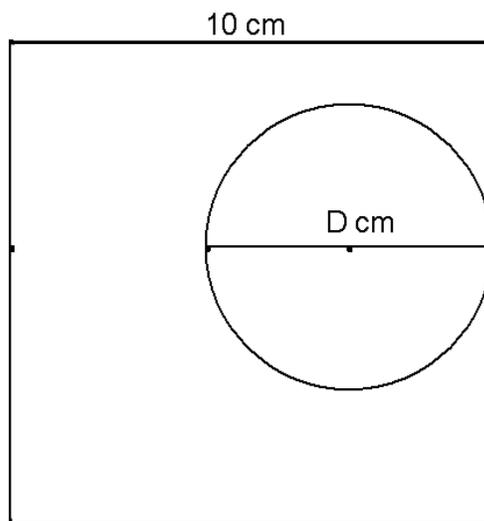


Figure 9-7

- Look at the center of the coin, marked by a red point. Imagine rolling the coin around the square so that it always remains just touching the edge of the square. Trace the square to make a copy of it. On a separate sheet, trace the circle and mark its center. Then cut out the circle and punch a small hole in the center. Place your circle in the square as shown in Figure 9-7, with a pencil in the hole at its center. Move the circle slowly around the edge of the square and let the pencil trace out the location of the center as the circle moves. Move the circle all the way around the square. What shape did the pencil trace out on its voyage?
- We will call the square traced out by the pencil "the winner's square," since coins whose centers fall into this square are winners in the coin toss game. Measure to find the dimensions of this square, then fill out the first row in the table below. In the last column write the ratio of the area of the winner's square to the area of the 10cm square.

D= coin diameter	S=length of a side of the winner's square	A=area of winner's square	A/100
D			

- Fill in the second and third rows of the table, choosing circle diameters greater than 0cm and less than 10cm. Cut out new circles if you need to or re-label your work from #6 and #7 with the new dimensions. Look for patterns in the table. Use the patterns you find to fill out the last row of the table with D for the diameter. Since you do not know the value of D , your answers in this column will be in terms of D .
- Note that the ratios in your last column will always be between 0 and 1, just like a probability! If you substitute your value of D from Question 3 above, what probability do you get? How close is it to the probability you chose in Question 1?
- Set up your HP 39gs to use your probability expression to find the diameter of your coin so it matches the probability you wanted as closely as possible. Write this diameter below.

Now you have a tool you can use to answer any question about the Coin Toss Game!

The Coin Toss Game: Teacher Notes and Answer Key

This activity requires students to make predictions using experimental and theoretical probability, use statistical representations to analyze data, use measures of central tendency and range to describe a set of data, and apply concepts of probability to make predictions.

This investigation is a good way to introduce students to the ideas of experimental and theoretical probabilities using a game that is commonly played in carnivals around the country. The **Coin Toss 1** Aplet is used to simulate as many tosses as you like of a coin of any diameter you like ($0 < D < 10$) on a grid of 10 cm squares.

Texas Essential Knowledge and Skills

The following TEKS are addressed in this activity. Examples of student actions in the course of the investigation for each TEKS are included.

- 6.9 The student uses experimental and theoretical probability to make predictions
 - Students use simulations to predict the size of coin needed
- 6.10 The student uses statistical representations to analyze data
 - Identify mean (using concrete objects and pictorial models, median, mode, and range of a set of data.
 - Students look at the distribution of probabilities from their trials
 - Solve problems by collecting, organizing, displaying, and interpreting data
 - Students organize the data from the simulations into a table and find the optimal coin size
- 7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events
 - Students model a popular carnival game
- 7.12 The student use measures of central tendency and range to describe a set of data
 - Make inferences and convincing arguments based on an analysis of given or collected data
 - Students argue the probability associated with a coin size
 - Describe a set of data using mean, median, mode, and range
 - Students use histograms and summary statistics to describe a set of trials in the coin toss game
 - Choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation
 - Optional, but teacher can choose to challenge students on this
- 8.10 The student applies concepts of theoretical and experimental probability to make predictions
 - Students predict the size of the optimal coin
 - Use theoretical probabilities and experimental results to make predictions and decisions
 - Select and use different models to simulate an event
 - Select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, box-and-whisker plots, histograms, and Venn diagrams, with and without the use of technology
 - Student creates histograms
 - Optional: students create box-and-whisker plots
- 1. The first step in the investigation is to have a discussion about the target probability of winning the game. The discussion will be as much about fairness as about making a profit for the club. The probability your class chooses must be less than 0.25, or they will not make any money; it must be greater than 0.01 or nobody will play. You may want to have the class make and fill out a table like the one below as they grapple with what probability is best. This table is not included with the Student Investigation sheet, as it is just a suggestion.

Probability	\$ Profit after 100 Tosses	\$/Toss
0.05	$100 * 0.25 - (0.05 * 100 * 1) = \20	\$0.20
0.10	$100 * 0.25 - (0.10 * 100 * 1) = \15	\$0.15
0.15	$100 * 0.25 - (0.15 * 100 * 1) = \10	\$0.10
0.20	$100 * 0.25 - (0.20 * 100 * 1) = \5	\$0.05
0.25	$100 * 0.25 - (.25 * 100 * 1) = \0	\$0.00

Your class may choose any value from 0 to 1. They may discuss how many people are likely to visit their game booth during the day of the fair, how many people will play multiple times, etc. Note that probabilities near 0.15 give a profit of a dime a play and still allow close to one in six players to win.

2. Divide the students into small groups and assign each 1 or 2 diameters to investigate using the **Coin Toss 1** Aplet. Ask to see their histograms after 10 trials. Note that students should choose trials of at least 30 tosses. If there is time, you can ask two groups to investigate the same diameter, but ask them to use different numbers of tosses. Then ask them to look at the differences in their histograms (see Extension 3).
3. The table below shows the theoretical probability of winning the game with a coin of the given diameter. Draw the table on the board and let the students fill in each row with their experimental results. Look for results that are not roughly in line with the theoretical results; there may be a problem (or an opportunity) with the group's data.

Diameter	Probability
10	0
9	0.01
8	0.04
7	0.09
6	0.16
5	0.25
4	0.36

4. The square does measure 10cm x 10cm when printed on 8.5"x11" paper. The diameter of the coin is approximately 6cm.
5. The Table 9-8 below shows the "winner's" square.

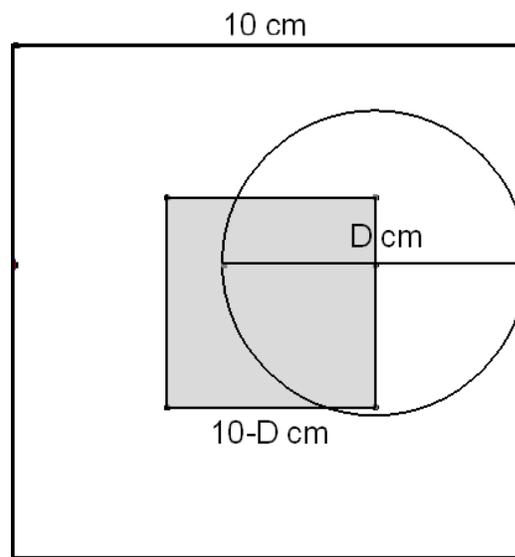


Figure 9-8

- 6-7. The table below has the first row filled out. The 2nd and 3rd rows have sample sizes supplied. Your students may have chosen other sizes.

D= coin diameter	S=length of a side of the winner's square	A=area of winner's square	A/100
6cm	4cm	16cm ²	16/100=0.16
3cm	7cm	49cm ²	49/100=0.49
9cm	1cm	1cm ²	1/100=0.01

D= coin diameter	S=length of a side of the winner's square	A=area of winner's square	A/100
D	10-D	$(10-D)^2$	$(10-D)^2/100$

8. Hopefully, the theoretical probability will match pretty closely to the simulated probability.
9. There are two easy ways to use the HP 39gs to work on this problem. First, you can enter the equation $P = (10-D)^2/100$ in the Solve Aplet, then enter your target probability for P and solve for D. Second, you can enter $(10-X)^2/100$ in the Function Aplet and use the table to investigate values for X.

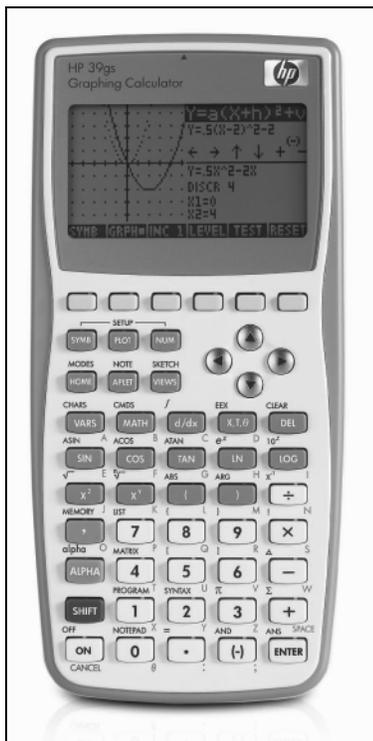
Extensions

You can extend this activity by changing any of the following fixed (but potentially variable) values:

1. The cost to toss a coin. What if it cost \$1 to play?
2. The cost of the prizes. What if it cost \$0.50 for each prize?
3. The number of tosses per trial. Examine the difference in the histograms for 10 trials with 5 tosses vs. 10 trials with 50 tosses, given the same diameter coin. There should be much more variability with 5 tosses per trial.

10 Quick Start Guide to the HP 39/40gs

Author: G.T. Springer



What are Aplets?

The simplest way to visualize an aplet is to think of it as a specialized room in which to work. For example, if you want to work with functions, then choose the Function Aplet. Do you want to test an inferential hypothesis? Choose the Inference Aplet. Each aplet provides tools and views that are particularly suited to its purpose.

The Aplet Library

Aplets are chosen from the APLET Library by pressing the **APLET** key. Simply highlight the one you want and press the **START** menu key or press **ENTER** (Figure 10-1).



Figure 10-1: Press the **APLET** Key!

Types of Aplets

There are three basic types of aplets. First, there are the core aplets that contain the majority of the functionality of the HP 39gs. These aplets are as follows:

- Function
- Parametric
- Polar
- Sequence
- Solve

- Statistics (for descriptive statistics)
- Inference (for inferential statistics)

These applets all have Symbolic (**SYM**), **PLOT**, and Numeric (**NUM**) views. The **PLOT** and **NUM** views each have a **SETUP** view controlling their appearance. These views are chosen so as to mirror the way in which most people work mathematically. Generally, most people will enter an equation (in the **SYMB** view) and then either plot it (in the **PLOT** view) or explore it in tabular form (in the **NUM** view).

The second set of applets consists of solvers for particular areas of study, specifically:

- Linear Solver
- Triangle Solver
- Finance (TVM Solver)

Third are the Explorer applets, designed to allow exploration of the connection between the graph of a function and the parameters in the symbolic expression of the function. These applets include:

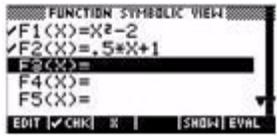
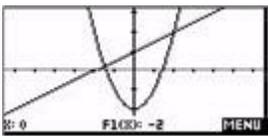
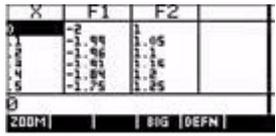
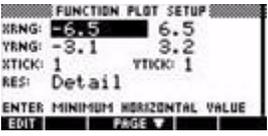
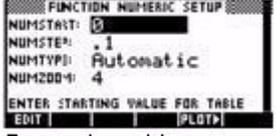
- Quad Explorer
- Trig Explorer

Four of the most commonly used core applets, illustrating their similar structure, are discussed below. Other applets may have very different views.

The Function Aplet

The Function Aplet contains all the features commonly used to explore functions, including plotting graphs of one or more functions, exploring tables of function values, and finding points of interest (roots, extrema, intersections, etc.). These features are presented in one or more views of the Function Aplet. By default, the Function Aplet starts in the Symbolic View. Table 10-1 shows the commonly used views of this Aplet and briefly indicates the purpose of each.

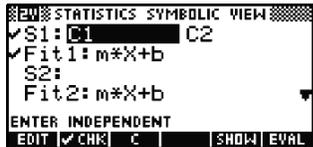
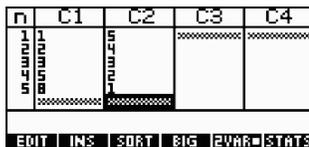
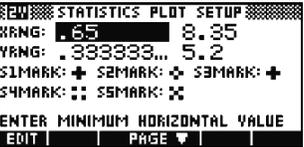
Table 10-1 Standard Views in the Function Aplet

View Name	SYMB (Symbolic)	PLOT (Graphic)	NUM (Numeric)
	 <p>The default view</p>	 <p>Plot function graphs</p>	 <p>See tables of values</p>
SETUP (Shift +)	No Separate View	 <p>Setup the window</p>	 <p>Setup the table</p>

The Statistics Aplet

The Statistics Aplet facilitates the display and analysis of numerical data with its descriptive statistics functionality. This applet has views similar to the function applet; notice, however, that the default view is the Numeric view. The views and brief descriptions appear in Table 10-2 below.

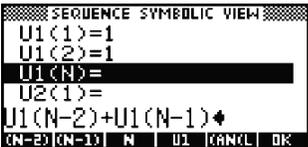
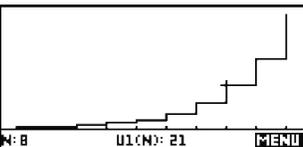
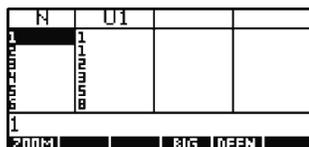
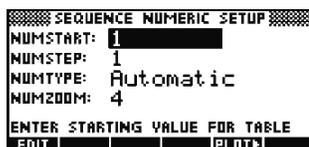
Table 10-2 Standard Views in the Statistics Aplet

View Name	SYMB (Symbolic)	PLOT (Graphic)	NUM (Numeric)
	 <p>Define an analysis</p>	 <p>Plot 1- or 2-var data</p>	 <p>The default view</p>
SETUP (Shift +)	 <p>Choose fits</p>	 <p>Setup the graph</p>	No Separate View

The Sequence Aplet

The Sequence Aplet helps students and teachers create and investigate sequences. Arithmetic, geometric, even the Fibonacci Sequence, are easy to create, graph, and tabulate. Again, this aplet has the familiar Symbolic, Plot, and Numeric views it shares with the other aplets. These views are summarized in Table 10-3.

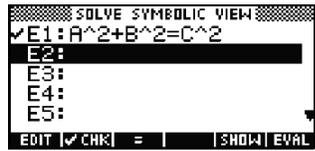
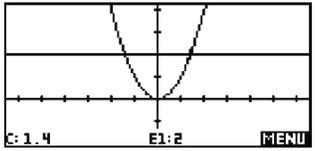
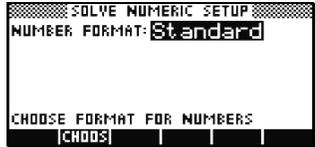
Table 10-3 Standard Views in the Sequence Aplet

View Name	SYMB (Symbolic)	PLOT (Graphic)	NUM (Numeric)
	 <p>The default view</p>	 <p>Plot stair-step or cobweb</p>	 <p>The table view</p>
SETUP (Shift +)	No Separate View	 <p>Setup the graph</p>	 <p>Setup the table</p>

The Solve Aplet

The Solve Aplet gives you the tools you need to find or approximate solutions to equations. You can enter up to ten equations, in as many variables as you like, however, you can only solve with one equation at a time. Again, this aplet has the familiar Symbolic, Plot, and Numeric views it shares with the other aplets. These views are summarized in Table 10-4.

Table 10-4 Standard Views in the Solve Aplet

View Name	SYMB (Symbolic)	PLOT (Graphic)	NUM (Numeric)
	 <p>The default view</p>	 <p>Plot 1- or 2-var data</p>	 <p>Solve for any variable</p>
SETUP (Shift +)	No Separate View	 <p>Setup the graph</p>	 <p>Select number format</p>

To solve any equation for one of its variables, first enter the equation in the Symbolic View. Then switch to the Numeric View and enter values for all of the variables whose values you know. You may also enter a seed value for the variable of interest. Finally, highlight the variable of interest and press the **SOLVE** menu key.

Save Your Own Aplets

You can save any core aplet with a filename of your choosing. You can also share these aplets with others, via cable or infrared connectivity. This means you can save your work, use the aplet for something else, and then come back to your work later. You can also set up an aplet, with window settings, table settings, etc. and send it to students so they all start on the same page without having to spend time entering those settings and other data that you want them to use for their explorations of mathematical topics.

Custom Teaching Aplets

There are literally hundreds of aplets created by teachers for use with specific topics. There are even some game aplets available for the HP 39gs. Some of these are just core aplets saved with data; others are highly programmed to address topics difficult to teach. You can find these aplets on the Web and download them via the Connectivity Kit. Go to www.hp.com/calculators to download the Connectivity Kit and the HP Custom aplets.

The HP 39gs User Interface

The user interface of the HP 39gs provides you with many ease-of-use benefits, including context-sensitive menus, input boxes, choose boxes, and check boxes.

Menu Keys

The top row of keys on the HP 39gs keyboard has no labels. These are the context-sensitive menu keys. These keys take their functionality from the list of functions printed at the bottom of the display. There are six menu keys in all. Figure 10-2 below shows the menu key functions as they appear in the Plot View of the Function Aplet. The functions accessed here via the menu keys are (from left to right) the **ZOOM** menu, the **TRACER** (an on/off toggle), **GOTO**, the **FCN** menu, **DEFN**, and **MENU**. Pressing the menu key below one of these labels opens the menu or performs the function indicated. These functions change as you move from one view or context to another.

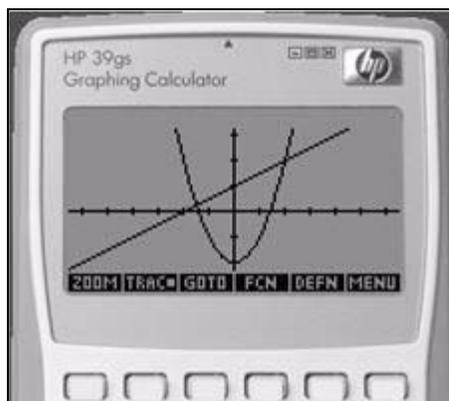


Figure 10-2: The Menu Keys

Input Boxes

The most common user interface device is the *Input* box. Figure 10-3 shows the Function Aplet's Plot Setup menu, with the **Minimum Horizontal Value** highlighted. The value, **-6.5**, sits in an input box. You can edit the current value by pressing the **EDIT** menu key, or enter any acceptable value in this box simply by typing it. Figure 10-4 shows the user entering a value of zero in the input box. Notice that editing occurs at the bottom of the screen rather than in the field itself. This manner of entry was chosen because the fields can be somewhat small, and users often want to enter complex expressions rather than just a few digits. Using the entire bottom of the display for entry gives you a larger typing area and lets you see the whole expression. This approach is used consistently in all aplets, so it is easy to get used to.



Figure 10-3: Plot Setup Menu

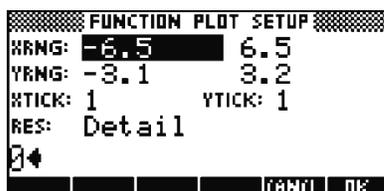


Figure 10-4: An Input Box



Figure 10-5: The New Value

For another example, the **Function Symbolic View** uses input boxes for the 10 functions you can enter to graph or tabulate. Figure 10-6 shows the **Function Symbolic View** with the 1st function input box highlighted. As in the previous example, you can either press the **EDIT** menu key or just start typing. Figure 10-7 shows the user entering **F1(X)=X² - 2**. Note that the expression is entered at the bottom of the screen, not in the **F1(X)** field itself. While being edited, the expression appears at the bottom of the screen; it is moved to the proper field when the **ENTER** key is pressed. Figure 10-8 shows the results when the user presses the **OK** menu key (or **ENTER**).

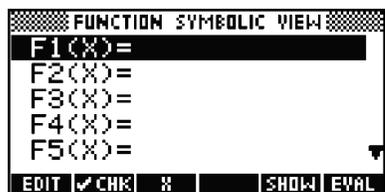


Figure 10-6: The Symbolic View

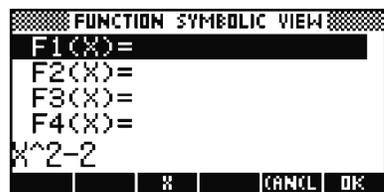


Figure 10-7: Editing F1(X)

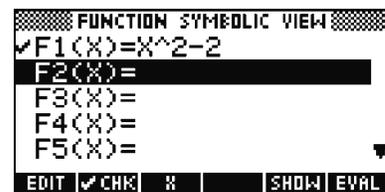


Figure 10-8: The Input Box

Choose Boxes

Some of the menus use *Choose boxes* to present you with different options for one or more fields in the menu. Figure 10-9 shows the Mode Menu with the **ANGLE MEASURE** field highlighted. The second menu key now has a label that says **CHOOS**. Pressing the **CHOOS** menu key opens the Choose box for this field, with options to choose **Degrees**, **Radians**, or **Grads** (gradients). See Figure 10-10. Use the arrow keys to make your selection and then press the **OK** menu key (Figure 10-11). Throughout the core aplets, the **ENTER** key is identical to the **OK** menu key, and the **ON** key is identical to the **CANCEL** menu key.



Figure 10-9: : Mode Menu



Figure 10-10: A Choose Box

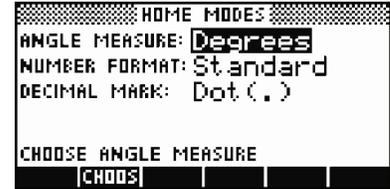


Figure 10-11: Switch to Degrees

Check Boxes

Some of the menus utilize check boxes to toggle an option off and on. Figure 10-12 shows the Function Plot Setup menu's second page. All the options here (drawing axes, drawing a grid, showing labels, etc.) are controlled via check boxes. The second menu label shows **CHK** with a check mark beside it. With any check box field highlighted, press the **CHK** menu key to enable/disable the selected option. Figure 10-13 shows the **Draw Axes** option being disabled; pressing **PLOT** now will show the graphs without the axes drawn (Figure 10-14).



Figure 10-12: Check Boxes



Figure 10-13: Disable Axes

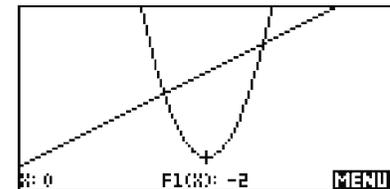


Figure 10-14: No Axes Drawn!

Entering and Editing Data

There are a few helpful hints you should know as you enter, edit, and delete data in the core aplets:

To begin with, you can store and retrieve numerical data in the variables **A** through **Z**. But first, you have to know how to enter letters of the alphabet. These letters appear near the bottom right corner of most of the keys on the HP 39gs keyboard, and are accessed by pressing the **ALPHA** shift key first. For example, press **ALPHA+VARS** to enter the letter **A**. You do not need to hold down the **ALPHA** shift key - just press and release it.

While entering data in the entry line, you can use the **DEL** key to delete any character. If the insert cursor is at the end of an entry, pressing **DEL** will delete the last character. In Figure 10-15, pressing the **DEL** key will delete the **2**.



Figure 10-15: Edit cursor at end

In Figure 10-16, the insert cursor has been moved over the **5** in the same expression. Now pressing **DEL** will delete the **5**. Note that the shift of the **DEL** key is **CLEAR**. Use **CLEAR** to clear the entire entry line. There is a shortcut to this as well. Press the **ON** key to cancel any entry (it says **CANCEL** below the **ON** key).

DEL and **CLEAR** have other uses that are consistent across the core aplets.



Figure 10-16: Cursor on the 5 now

In Figure 10-17, the minimum horizontal graphing value (**-2**) is highlighted. Pressing **DEL** here will reset that value to the default (**-6.5**).

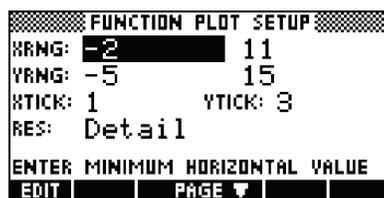


Figure 10-17: Plot Setup Values

CLEAR will reset all the values in this menu to their defaults, as in Figure 10-18.



Figure 10-18: Reset the Values

In Figure 10-19, the cursor is in **C 2** of the Statistics Aplet. Press **CLEAR** now to see the chose box where you can select to clear either the selected column or all the data.



Figure 10-19: Clearing Data

Figure 10-20 shows a similar use of **CLEAR** in the Function aplet.

These uses of **DEL** and **CLEAR** are consistent across all of the core aplets with respect to restoring default values and clearing data or expressions.

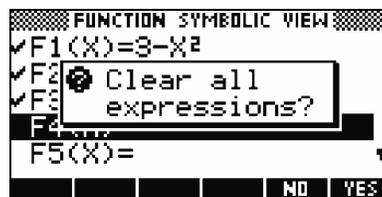


Figure 10-20: Clearing Functions

The Structure of Aplets

All core aplets have four views that they treat in a similar fashion. Three of these views are based on symbolic, graphic, and numerical views of mathematical objects. The fourth view is called the *Home* screen, where calculations are performed. Figure 10-21 illustrates the keys that give the user access to the four views. The **SYMB** key opens the Symbolic view of an aplet. The **PLOT** key opens the Graphic view, while the **NUM** key opens the Numerical view. Note that the shift of each key opens a setup menu to control each view. For example, instead of a Window view, there is a Setup Plot view. There is a **HOME** key to give you access to the Home screen. Note the shift of the **HOME** key gives you control over the calculator modes.



Figure 10-21: The SYMB, PLOT, and NUM views

The default aplet when the HP 39gs is powered up is the Function Aplet. Notice the current aplet name is shown in the display header. In this chapter, we will use the Function aplet to model the four views and their operations.

The Home View

The HP 39gs Home screen is divided into four parts, from top to bottom. There is a **Header** at the top, with the name of the aplet you're currently using and whether you are in Degree- or Radian-Mode. Next comes the **History** of calculations. Below that is the **Entry Line**. Finally, at the bottom are the **Menu Labels** for the menu keys. See Figure 10-22 for an illustration of these parts of the Home Screen.

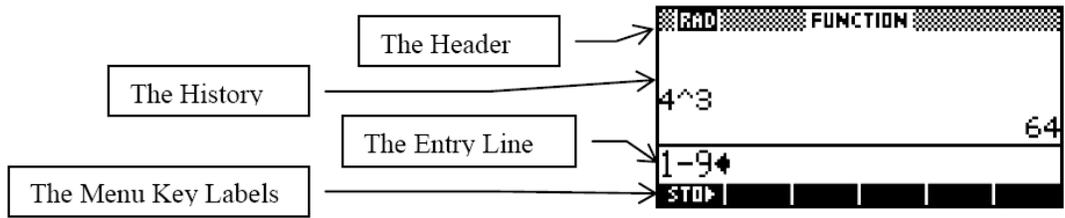


Figure 10-22: The Home Screen

Calculate the square root of $2/3$ by entering that expression and pressing the **Enter** key. The HP 39gs displays up to 12 digits of accuracy (Figure 10-23).

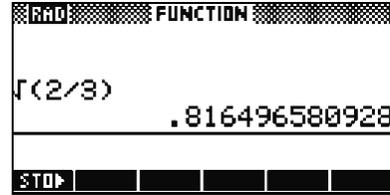


Figure 10-23: A calculation

Now use the up-cursor key to highlight the expression you entered (see Figure 10-24). Note that two additional menu labels pop up: **COPY** and **SHOW**. The **SHOW** command typesets the expression using standard 2-D mathematical notation. This command is useful for students who want to check that the expression they entered - with one or more sets of parentheses - is exactly what they wanted to enter. If they entered the expression incorrectly, they can use the **COPY** command to fix it. The **COPY** command puts a copy of the expression in the entry line.



Figure 10-24: COPY and SHOW

Try it out by pressing the key below the **SHOW** label. Your display should look like Figure 10-25.



Figure 10-25: The Show command

Press the **OK** menu key when you are done viewing the expression, then press the **COPY** key to see the expression pasted into the entry line as in Figure 10-26. If the student had meant to enter the square root of $2/5$, he or she can fix it by moving the insert cursor over the **3**, deleting it, and inserting **5**. Press the **ON** key to cancel any entry.

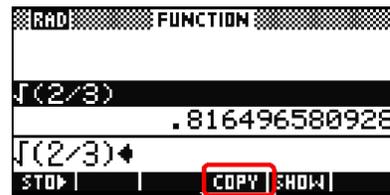


Figure 10-26: The Copy command

Students can use the History to show all of their work as they solve problems. If they make a mistake, they can delete lines from the History by using the **DEL** key. Note that the shift of the **DEL** key is **CLEAR**. Use the **CLEAR** command to delete the entire history. But be careful - there is no undo for this action!

The SYMBOLIC View

The Symbolic View is used to record and manage symbolic representations, including functions, parametric equations, and sequences.

With the Function Aplet active, press the **SYMB** key to access the set of 10 function definitions, F1(X) through F9(X) and F0(X). The last definition used is highlighted by default. Figure 10-27 shows the Function Symbolic View with no functions defined and **F1(X)** highlighted.

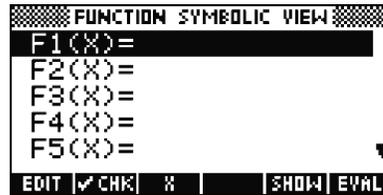


Figure 10-27: Symbolic View

Note the new menu labels. **EDIT** opens an input box to edit the chosen definition, **F1(X)**. **X** is just a typing aid, and **EVAL** will simplify an algebraic expression to some extent.

You do not have to press the **EDIT** key to start a definition if there is not one already there. Just start typing and the input box will open. Enter the expression $3 - X^2/2$, as in Figure 10-28, then press the **Enter** or **OK** key.

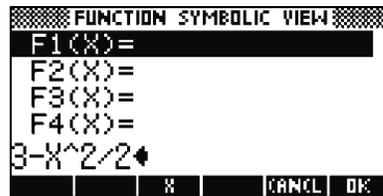


Figure 10-28: Define F1(X)

The expression is pasted into the **F1(X)=** field and the highlight bar moves down to **F2(X)**. Look at Figure 10-29, which shows the completed **F1(X)** definition.

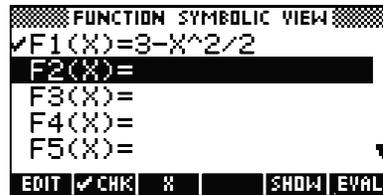


Figure 10-29: F1(X) is checked

Note a check mark appears to the left of the **F1(X)** definition. The mark indicates that **F1(X)** is now active for graphing. You can use the **CHK** menu key to enable or disable any function for graphing or tabulating, without erasing the function definition.

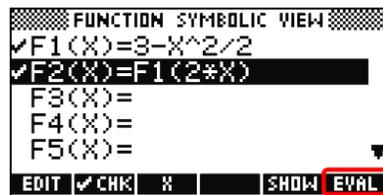


Figure 10-30: Define F2 via F1

To see how **EVAL** works, enter the expression **F1(2X)** in **F2(X)**, as in Figure 10-30. Highlight **F2(X)** and press the **EVAL** menu key to see the expression changed to **3-(2x)2/2** (Figure 10-31).

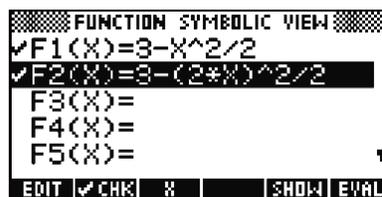


Figure 10-31: Use EVAL

The Symbolic view of the core applets work in a similar fashion. For example, the Sequence Applet Symbolic view lets you define up to 10 sequences, while the Symbolic view of the Solve Applet contains definitions for up to 10 equations.

The PLOT View

The Plot View shows you the graphic view of the mathematical objects in the active core applet. Here you can trace along a graph, find intersection points, etc.

Press the **PLOT** key to see the graph in the current window (Figure 10-32). If your graph does not look like Figure 10-32, then press the Shift of the **PLOT** key (Plot Setup) and adjust the Plot Setup menu to agree with Figure 10-33.

The default settings in the Plot Setup menu require a little explanation. The HP 39gs display has 131 pixel columns, which allow the cursor to make 130 movements from one side of the display to the other. Setting the domain (**XRNG**) from -6.5 to 6.5 forces each cursor movement to represent 0.1 units.

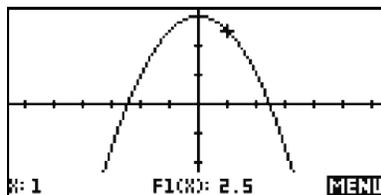


Figure 10-32: The Graph

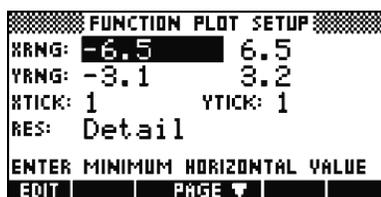


Figure 10-33: Plot Setup menu

Likewise, the display has 64 pixel rows for 63 cursor moves; thus, setting the **YRNG** from -3.1 to 3.2 forces each vertical pixel move to represent 0.1 units as well. Notice there is a new menu label here.

Press the **Page** ∇ key to see the second page of the Plot Setup menu. Here you will find further options to control how the graphs are displayed and formatted.

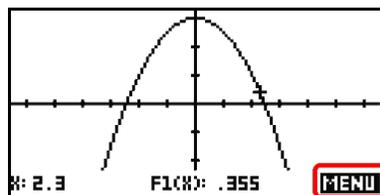


Figure 10-34: Root near $x=2.3$

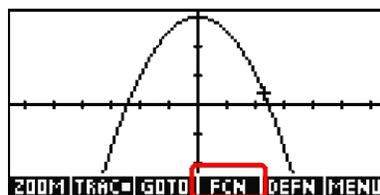


Figure 10-35: The FCN folder

Once your Plot Setup menu agrees with ours, press the **Plot** key to see the graph of $F1(X)$. By default, the Tracer is on. Press the left- and right-cursor keys to trace along the function. Figure 10-34 shows the tracer near one of the roots of $F1(X)$ near $X=2.4$. Press the **MENU** key to see **ZOOM**, **TRACE**, **GOTO**, **FCN**, **DEFN**, and **MENU**. Each of these new commands relates directly to the graphs of functions. For now, press **FCN** (Figure 10-35) and choose the **ROOT** option (Figure 10-36) to see a 12-digit approximation of the root, at $X=2.44\dots$ (Figure 10-37).



Figure 10-36: The FCN folder

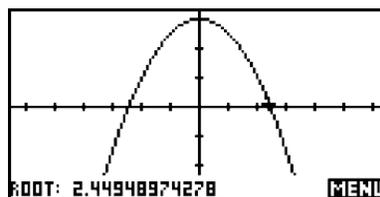


Figure 10-37: The Root

The NUMERIC View

The Numeric view gives you the tools you need to explore a function via an X/Y table. You can select the start- and step-values for the independent variable, zoom in and out on a row of the table, and navigate through the table values.

Press the **NUM** key to see the default table of values for our function $F1(X) = 3 \cdot X^2/2$ (Figure 10-38). Use the cursor keys to scroll about in the table.

X	F1		
0	0		
.1	.15		
.2	.6		
.3	1.35		
.4	2.4		
.5	3.75		

ZOOM BIG DEFN

Figure 10-38: Table of values

You can zoom in or out on a selected row of the table using the **ZOOM** menu key. For example, the current table has x-values that step by 0.1. Press the **ZOOM** menu key and select the **Zoom Integer** option (Figure 10-39) to make the x-values step by 1. Figure 10-40 shows the result. Of course, you can use the Numeric Setup menu (**Shift+NUM**) to change the start and step values of the table to be any values you like.

Out			
Decimal			
Integer			
Trig			
Un-zoom			

CANCEL OK

Figure 10-39: Table zooms

Press **Shift+NUM** to see the Numeric Setup menu, as shown in Figure 10-41. **NUMSTART** is the starting x-value for the table. **NUMSTEP** controls the step value for the x-values. Under **NUMTYPE**, you have two options: **Automatic** for creating a full table of values, or **Build Your Own**. The latter gives you an empty table, lets you type in x-values, and supplies the corresponding y-values. **NUMZOOM** controls the zoom scale factor for zooming in and out on a row of the table. By default it is set to 4.

X	F1		
0	0		
1	1.5		
2	6		
3	13.5		
4	24		

ZOOM BIG DEFN

Figure 10-40: Zoom Integer

Finally, note the **PLOT>** feature. If you press this menu key, the table values will correspond exactly to the tracer values in the **PLOT** view.

Because the core applets all utilize the same basic structure and design, you are now ready to explore the other applets on your own!

FUNCTION NUMERIC SETUP	
NUMSTART:	0
NUMSTEP:	1
NUMTYPE:	Automatic
NUMZOOM:	4
ENTER STARTING VALUE FOR TABLE	
EDIT	PLOT

Figure 10-41: Numeric Setup

www.hp.com/calculators

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