

# Mathematical Exploration with HP Prime



## Sometimes True, Always True, Never True

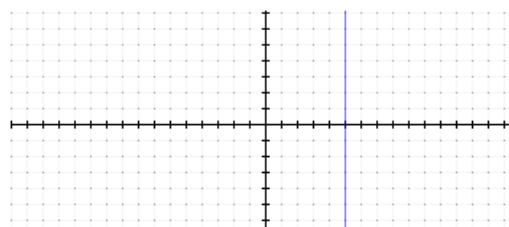
If we look at the statement  $3x + 1 = 16$  we can say immediately that this is true only when  $x = 5$ . Compare that to the statement  $2x + 3x = 5x$ . That is true for any value of  $x$ . It doesn't matter what  $x$  is, 2 times a number plus 3 times the same number is always the same as five times that number. Now compare with the statement  $\sin(x) = 2$ . There is no value of  $x$  where  $\sin(x) = 2$  because the values of  $\sin(x)$  are always between negative and positive one.

So:

- $3x + 1 = 16$  is sometimes true ... in fact true in one instance  $x = 5$
- $2x + 3x = 5x$  is always true ... it is true for all  $x$
- $\sin(x) = 2$  is never true.

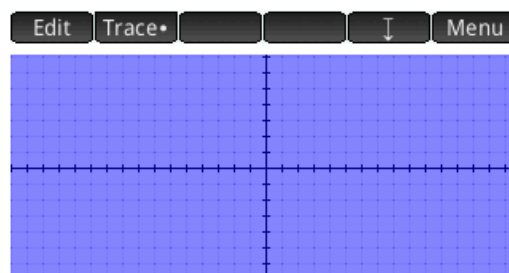
See what happens when we type these statements into the Advanced Graphing App in HP Prime.

1. Click Apps
2. Select Advanced Graphing
3. Type the statement into the line for S1
4. Click Plot
5. Press Menu then Defn (you can drag the graph into a more suitable position if you like)
6. Click Symb to edit the statement, press backspace to delete, then type the next statement and continue as before.



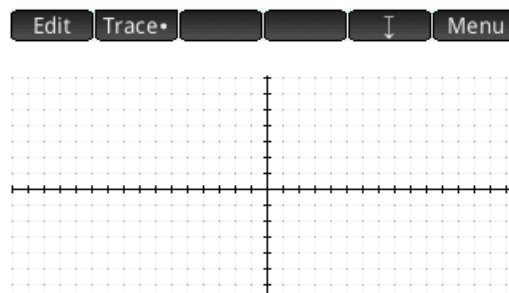
S1:

$$3 * X + 1 = 16$$



S1:

$$2 * X + 3 * X = 5 * X$$



S1:

$$\sin(X) = 2$$

The graph for  $3x + 1 = 16$  is a graph of  $x = 5$ . These two statements are the same. The solution is  $x = 5$

The graph of  $2x + 3x = 5x$  is everything. It is always true.

The graph of  $\sin(x) = 2$  is nothing. It is never true.

Mathematically we are very interested in situations of the first two types. An equation which is always true is called an **identity**. Essentially the statements are identical. Equations which are sometimes true have **solutions** and we are interested to know which equations have solutions and how many they might have.

In both cases we can take the statement and manipulate the algebra to see if we can find statements which are solutions or are identical.

For  $3x + 1 = 16$  the solution is  $x = 5$

But  $2x + 3x = 5x$  is always true so it is an identity. We write it:  $2x + 3x \equiv 5x$

**Note:** Just because the screen shows all, some or none shaded does not prove that these are the only outcomes, just in the range that the screen is showing. You could zoom out (press the  $-$  key a few times, which is more convincing, but you cannot see an infinite range!) Nonetheless this gives a very good visual indication.

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## Activity 1

Test these statements with the Advanced Graphing App. Are they sometimes, always or never true? In each case, if it is sometimes true, what is/are the solution(s)? If it is always true, then the statement is an identity, so re-write it with the identity symbol. If it is never true, explain why.

1.  $2\sin(x) = 2$  (remember the calculator is probably set to radians)
2.  $x^2 = x + 6$
3.  $x^2 = x - 6$
4.  $\frac{1}{x+2} = 0$
5.  $\frac{1}{x+2} = 1$
6.  $5x(x - 3) + 2(x + 1) = 5x^2 - 13x + 2$
7.  $3xy - 2yx = xy$
8.  $x^2 - y^2 = (x + y)(x - y)$
9.  $x^2 + 6 = y - 5x$
10.  $(x - 1)^2 = 4 - y^2$

**Note:** In written algebra when we write  $xy$  we mean  $x \times y$ . However,  $xy$  is a perfectly good name for a single variable and so there is a possible confusion. In computer systems you should always type  $x \times y$  i.e.  $X*Y$

We hope that you noticed that with one variable (just  $x$ ), 'sometimes true' gave one or more solutions. However, with both variables involved ( $x$  and  $y$ ) 'sometimes true' gave a relationship. One was a circle, one was a parabola. With one statement on its own we have three types of outcome:

1. An equation with (or without) solutions;
2. An identity;
3. A relationship (most generally called mapping).

## Activity 2

Use the advanced graphing App to construct your own examples in each of these three categories.

Equation		Identity	Mapping
With solution(s)	Without solution(s)		

Extend the table as you explore.